Sonar false alarm rate suppression using classification methods based on acoustic modelling

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Abstract

The use of high-resolution, active sonar systems in littoral environments often results in high false alarm rates. False alarm rate inflation (FARI) and non-Rayleigh reverberation (NRR) are two well-documented causes. FARI may occur when the reverberation in the normaliser window is non-stationary, while NRR may occur when the sonar footprint is too small for the central limit theorem to apply for the scatterer statistics. The main originator for false alarms in littoral environments are either the sea floor itself or objects located on the sea floor.

Automatic classification methods may be used to reduce the false alarm rate. Conventionally, advanced sonar processing or image processing techniques have been used directly on received data. Increased availability of environmental information allows for more sophisticated algorithms that employ acoustic modelling to extract more information from recorded data.

This thesis addresses two topics of interest. The first topic is on how acoustic modelling combined with environmental knowledge may be used to increase the ability of anti-submarine warfare sonars to classify a detected target. The second topic is on how environmental uncertainty may be reduced in order to improve the fidelity of the proposed classification algorithms.
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Summary

Littoral sonar operation is complicated by high false alarm rates, particularly for high–resolution sonar systems such as modern anti–submarine warfare sonars. Typically, high concentrations of false alarms are observed around wrecks, sea mounts, and along rocky ridges. Some of these false alarms result in generation of tracks exhibiting target–like behaviour. Improved classification algorithms are needed to reduce the false alarm rate.

This thesis presents seven papers describing methods that may increase the performance of modern sonars in anti–submarine warfare operations. All presented methods pursue at least one of two topics of interest. Topic A is on exploiting available environmental information to increase the classification ability of anti–submarine warfare sonars in littoral waters. Topic B is on limiting the environmental uncertainty during operation.

Four papers relate to topic A and introduce new methods for target classification. The two first papers present methods on how reverberation modelling may be used to predict what areas are prone to increased false alarm rates. The next two papers apply a ray tracing algorithm for determining the depth of a detected target. This algorithm requires a known environment and recorded arrival times and angles. The fourth paper also includes a focalisation method that tunes the environment in order to increase the accuracy of the target depth estimates. All these methods are sensitive to environmental uncertainty. Errors in environmental parameters, particularly the sound speed profile, may result in ambiguous or erroneous target classification. The remaining papers, all on topic B, seek to alleviate this problem. The fifth paper introduces a method that inverts sound speed profiles from data recorded on available sensors, such as measurements of sound speed close to the sonar depth. The sixth paper presents a method that classifies areas as acoustically stable or unstable on basis of modelled oceanographic data by employing empirical orthogonal function classification, clustering approaches, and acoustic modelling. The seventh paper investigates how the sonar – target geometry influences acoustic stability in face of environmental uncertainty.
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Paper 2  Predicting sonar false alarm rate inflation using acoustic modeling and a high-Resolution terrain model

Paper 3  Target depth estimation using a ray backpropagation scheme on sonar data – simulations and experiments

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Paper 5  Inverting the water column sound speed

Paper 6  Finding acoustically stable areas through EOF classification

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1 Introduction

Sea trials in littoral environments show that high-resolution, active sonars generate particularly many false alarms in presence of ship wrecks and terrain features such as seamounts and underwater ridges [1–4]. Possible causes for high false alarm rates include false alarm rate inflation [5–8] and non-Rayleigh reverberation [1, 9–16]. False alarm rate inflation is induced by a non-stationary reverberation power level in the normaliser windows. Non-Rayleigh reverberation, also called clutter, appears when the sonar resolution is so high that the sonar footprint is too small for the central limit theorem to apply for the scatterer statistics. The false alarm rate depends on the sonar system, the choice of signal processing, and on the present environment [1, 6, 13, 15, 16]. In littoral environments the main originators of false alarms are either the sea floor itself or objects located on the sea floor [2–4].

Recently, much research has been made on predicting and controlling false alarm rates. This research includes fields such as normalisation [6, 7, 11, 17], detection [10], image processing [18–20], acoustic modelling [4, 21, 22], and alternative matched filtering techniques [3]. The research is partly motivated by the increased use of high resolution naval sonars in littoral environments. After the cold war the focus of anti-submarine warfare has shifted from open water scenarios to littoral scenarios. Concurrently the resolution of naval sonars has improved; higher bandwidths due to improved processing and sonar technology, and improved bearing resolution due to the introduction of active towed array systems in naval warfare.

This thesis presents false alarm rate reduction methods based on acoustic modelling. False alarms are suppressed by classifying received sonar echoes as either false alarms or potential targets. The first four papers in this thesis present two different types of target classification methods. The first type of method predicts the occurrence of false alarm rate inflation using reverberation modelling. The predictions are used to estimate a probability of a received echo being a false alarm. The second type of method uses a ray back-propagation scheme [23] to estimate the depth of a detected target. Since false alarms typically originate from the sea floor, target depth information is a useful classification clue. The fidelities of the proposed methods depend strongly on the accuracy of available environmental input, since the acoustic field is sensitive to environmental uncertainty [24–29]. For mid-frequency sonars (1–10 kHz) an uncertain sound speed profile is an important contributor to the uncertainty in acoustic field predictions. The last four papers
of this thesis explore how uncertainty in sound speed may be reduced to increase the accuracy of acoustic modelling and thereby improve both the results of the proposed classification methods and sonar performance modelling in general.

1.1 Motivation

The Norwegian navy is procuring modern anti–submarine warfare (ASW) frigates, Fridtjof Nansen–class frigates. The first frigate was delivered to the Norwegian navy in 2006. The frigates are equipped with advanced acoustic sensors for submarine detection; a hull–mounted sonar called Spherion MRS2000 and an active towed array sonar called CAPTAS. Both systems have high resolutions and similar systems have been shown to generate large amounts of false alarms in littoral environments [4,22].

The Norwegian government funds a large number of programs intended to increase the knowledge of the environment in the Norwegian economic zone. In the underwater domain, these programs include oceanography (the Norwegian Meteorological Institute, Nansen Environmental and Remote Sensing Center, Institute of Marine Research), fishery (Institute of Marine Research), and sea floor mapping (Norwegian Mapping Authority, Geological Survey of Norway, Norwegian Defence Research Establishment). Sea floor mapping includes mapping of bottom depths and bottom properties. The ongoing project Mareano [30] gives public access to some of this information. The abundance of information on the subsurface environment may give Norway a considerable tactical advantage in ASW operations in Norwegian territory. However, knowing the environment is not enough, equally important is knowing how to take advantage of this knowledge. The Russian admiral Sergei Gorshkov said: ”The major navies of the world are technological equals – that navy possessing a superior knowledge of the environment, and knowing how to take tactical advantage, will be victor.” Gorshkov makes a point of not only knowing the environment, but also taking advantage of this knowledge. The latter is a key–point for the motivation of this thesis. The thesis includes seven papers that present methods and theories on how environmental knowledge may be extended and exploited with the overall goal to improve the classification performance of modern naval sonars.
1.2 Focus of the thesis

The main focus of the presented work is to improve the classification ability of active ASW sonars in littoral environments. The two main topics of this thesis are:

A How to exploit available environmental information in order to increase the classification ability of ASW sonars

B How to deal with environmental uncertainty

Topic A includes use of various sources of environmental information, refined processing algorithms, and acoustic modelling in order to extract additional information from recorded sonar data during operations. Topic B includes methods that reduce the impact environmental uncertainty has on the methods introduced in topic A.

1.3 History of naval sonars

The emergence of submarines in naval warfare introduced a need for new sensors capable of detecting submerged targets. Unlike electromagnetic signals, acoustic signals may propagate for hundreds of kilometers in water and still be detected above ambient noise. As early as 1490, Leonardo da Vinci showed that distant ships could be heard by extending a listening tube into the water by placing an ear to the dry end of the tube. During World War I the British developed the ASDIC, the first anti-submarine warfare (ASW) sonar. The first operational ASDIC, presented in 1919, was an active, hull-mounted sonar and operated on frequencies from 20 to 50 kHz.

In the inter-war period, low-frequency acoustic signals were observed to propagate longer distances than high-frequency signals due to frequency-dependent attenuation. Another important discovery, made in the late 1940s by the American scientist Ewing, was that submarines could be detected via other propagation paths than the direct path, e.g. bottom reflected path and in convergence zones. These two discoveries encouraged the development of high-powered, low-frequency, active sonar systems. New hull-mounted sonars were introduced with increased power and lower frequencies, culminating in the scanning sonar. The scanning sonar consisted of an array of
hydrophones capable of both transmission and reception. This allowed directionality in both transmission and reception.

Sonar development was clearly driven by the rapid development of the submarine. During the cold war, passive, towed array sonars were introduced as an alternative to active, hull–mounted sonars for long–range detection. However, as submarines became more quiet, passive sonar systems no longer obtained the necessary detection ranges, and as the speed of the submarines increased, longer detection ranges were required. In the 1980s and 1990s the low–frequency, active, towed array sonar was developed as an alternative to passive towed array sonars.

ASW in World War II consisted mainly of protecting high–value units, for instance transport vessels, from submarine attacks. Allied transport convoys were constantly attacked by German submarines, and ASW vessels were deployed to defend the convoys. During the Cold War, ASW was played out in the blue ocean with the intent of tracking large nuclear submarines at long distances. After the Cold War the focus of ASW has shifted from blue ocean scenarios to littoral scenarios. For instance protecting landing crafts during landing of military personnel or equipment. In littoral environments modern sonar systems, such as low–frequency, active, towed array sonars are troubled with high false alarm rates. In addition, nuclear submarines are no longer the main subsurface threat. Affordable, small, silent submarines with low target strengths are considered the main threat in the future. Their manoeuvrability enables them to hide in shallow waters close to the coast line among rocks and sea mounts, making them almost impossible to detect and classify.

1.4 Organisation of the thesis

Chapter 2 contains the necessary background for understanding the attached papers. Important concepts and theory are described and presented. Chapter 3 briefly summarises the key points of each attached paper. Chapter 4 gives an overall conclusion of the work described in this thesis. The attached papers are included at the end of the thesis.
2 Background

This thesis covers a range of subjects within the fields of acoustics and signal processing such as sonar processing, acoustic modelling, empirical orthogonal functions, inversion, detection theory, and sensitivity analysis. The following sections provide the reader with definitions of concepts and a background for each of these subjects.

2.1 Sonar systems

This thesis treats two different types of sonar systems; anti-submarine warfare (ASW) sonars and echo sounder systems. Each system is described in the following sections.

2.1.1 Anti–submarine warfare sonars

This thesis includes data from two different types of active ASW sonars; hull–mounted sonars and towed array sonars. Active ASW sonars use underwater sound propagation to detect submerged targets. An acoustic signal is transmitted from a source, the signal echoes off targets present in the ocean before returning to the acoustic receiver. At the same time ambient noise and returns from other acoustic scatterers in the ocean are received. The sonar attempts to detect the target echo among reverberation and noise.

Towed array systems are horizontal arrays of hydrophones towed by a vessel. Active systems also include a towed source. The main advantages of such systems are that the array depth is adjustable and that their large size allows high horizontal angle resolution even for low frequencies. Adjustable depth allows the sonar operator to place the array at a depth that is advantageous for long-range acoustic propagation, e.g. in a sound channel. Long arrays allow beamforming with high resolution in angle, which is essential for resolving target location at long ranges and also improves noise suppression, see section 2.2.1. Linear arrays, arrays with a single hydrophone width, have left–right ambiguity. This means that the array is unable to discern which side of the array a detected target is present. In modern ASW towed array systems this problem is solved by using either two parallel linear arrays or triplet arrays [31].

As the name suggests, hull–mounted sonars are fixed to the hull of the vessel with an acoustically transparent sonar dome separating the transducer-
ers from the water. Due to the size–limitation of such systems, they are limited to mid– or higher frequencies. Modern hull–mounted systems are able to beamform both vertically and horizontally and have no left–right ambiguities. Vertical beamforming is used to concentrate transmitted power in vertical angles dominated by propagating modes, e. g. propagation paths with minimal bottom interaction. Horizontal beamforming improves the capability of the sonar for locating a target.

2.1.2 Echo sounders

Echo sounding is a technique for measuring bottom depth by transmitting an acoustic pulse vertically from a hull–mounted echo sounder and receiving the subsequent bottom reflections. The bottom depth, $D$, is estimated using the following expression:

$$D = \frac{T}{2\hat{s}}.$$  \hspace{1cm} (1)

where $T$ is the time from transmission to reception, $\hat{s}$ is the depth–averaged slowness. Slowness is defined as:

$$s = \frac{1}{c},$$  \hspace{1cm} (2)

where $c$ is the sound speed. The sound speed must be measured frequently during operation in order to secure high accuracy in the measurements.

The coverage of an echo sounder system may be improved by using multi–beam systems. Multi–beam systems transmit an acoustic pulse in a wide sector and receive in beams with different vertical steering angles. For more information on beams, see section 2.2.1. Cross–path coverage may be gained by using ray tracing models to estimate cross–path bottom depths from arrivals at angles other than the vertical. The high–resolution topography information used in several of the attached papers was measured using a multi–beam echo sounder.

2.2 Sonar processing

The main focus of the thesis is on ASW sonar systems, the sonar processing described below is the processing used for detecting a submarine using a frequency modulated (FM) pulse. Other pulses, such as the continuous wave
pulse requires different processing, but since the FM pulse is the only pulse shape considered in the attached papers, theory on different pulses are not included here. The main intent of conventional active ASW sonar processing is to detect and locate targets whose acoustic returns are embedded in reverberation and noise. The following paragraphs give a brief introduction to the different signal processing techniques used in conventional sonar processing to detect and locate a target.

2.2.1 Beamforming

Beamforming is a commonly used technique to aid in detection and localisation of a target. Beamforming exploits the relative time delays of received arrivals on different hydrophones in the receiver array in order to determine the direction from which an echo arrives [32]. A conventional beamformer sums the received signal from all hydrophones after applying a phase shift that depends on hydrophone location. For linear arrays with $K$ equally spaced hydrophones the beamformed signal, $s_b$, equals [33]:

$$s_b(j, \theta) = \sum_{k=0}^{K-1} s_h(j, k) \exp \left(2\pi i k \frac{d}{\lambda} \sin \theta \right),$$

where $s_h(j, k)$ is the received signal in hydrophone number $k$ at sample number $j$, $\theta$ is the steering angle, $d$ is the distance between hydrophones, and $\lambda$ is the wavelength. On conventional sonars a set of beams with different steering angles are processed. The main advantages of beamforming are directivity and noise suppression [34]. Directivity allows determination of target direction. Noise sources are suppressed since the noise from only a limited angular space is received. An example of a beamformed data sequence is shown in Fig. 1. Interested readers are referred to Therrien [35] and Van Trees [36] for more thorough descriptions of beamforming.

2.2.2 Matched filter

Like beamforming, the matched filter is commonly used to improve detection and localisation of a target. Matched filtering [36], also called pulse compression, correlates the received signal with a known signal. The matched filtered
signal, $s_m(j, \theta)$, is then given by:

$$s_m(j, \theta) = \sum_{n=-\infty}^{\infty} h(j - n)s_b(n, \theta).$$  \hspace{1cm} (3)

For active sonars, $h(j)$ is a time–reversed version of the transmitted signal. The advantages of matched filtering include a processing gain and increased range resolution. The processing gain, $G$, depends on the product of the frequency bandwidth, $B$, and the pulse length, $T$, of the signal used:

$$G = BT.$$ \hspace{1cm} (4)

Note that increased pulse length also results in increased reverberation, and increased bandwidth in increased noise levels. An example of a matched filtered data sequence is shown in Fig. 1. Interested readers are referred to Therrien [35] and Van Trees [36] for more thorough descriptions of matched filtering.

### 2.2.3 Normalisation

After beamforming and matched filtering, the received signal is passed through a normaliser:

$$s_n(j, \theta) = \frac{s_m(j, \theta)}{n(j)},$$

where $j$ is the analysed sample, $s_n(j, \theta)$ is the normalised signal, also frequently called the estimated signal–to–reverberation and noise ratio (SNR), and $n(j)$ is the estimated background. The background is typically estimated as follows:

$$n(j) = \frac{1}{C} \sum_{l=-L/2}^{L/2} c(l) s_m(j, \theta),$$

where $L$ is the width of the normalisation filter. $C$ is defined as:

$$C = \sum_{l=-L/2}^{L/2} c(l).$$ \hspace{1cm} (5)
\( c(l) \) varies for different normalisers. For the CA CFAR (cell averaging constant false alarm rate) normaliser [5] \( c(l) \) is defined as:

\[
\begin{align*}
  c(l) &= 0, |l| < K \\
  c(l) &= 1, l \geq K
\end{align*}
\]

where \( K < \frac{L}{2} \). Samples where \( c(l) = 0 \) is commonly called the guard band [6].

The main intent of the normaliser is to remove trends from the received signal, such as the signal decay with range. The reader is referred to Richards [5] for thorough discussions on the subject of normalisation.

### 2.2.4 Detection

Target detection using active sonars is a binary decision problem, where the intent is to decide between two hypotheses:

1. a target echo is present in the received signal
2. a target echo is not present in the received signal

The decision is made by applying a threshold, \( T \), to the normalised signal [36]:

\[
\begin{align*}
  s_n(j, \theta) &\geq T \Rightarrow \text{choose hypothesis 1} \\
  s_n(j, \theta) &< T \Rightarrow \text{choose hypothesis 2}
\end{align*}
\]

Samples with a present target that results in a threshold–crossing are called detections. The normalised signal may fail to cross the threshold even with a target present. This is frequently called a miss. The probability of detection, \( P_d \), is the probability that the signal crosses the threshold when a target echo is present. \( P_d \) depends on the threshold and the signal strength distribution. Due to spikes in reverberation and noise, the normalised signal may exceed the selected threshold even without a present target. Such threshold–crossings are undesired and are called false alarms. Probability of false alarm, \( P_{fa} \), is the probability of undesired threshold–crossings. Assuming an exponentially distributed normalised signal and using a CA CFAR normaliser, then the probability of false alarm is given by [5]:

\[
P_{fa} = \exp(-T).
\]

The threshold is therefore uniquely determined from the desired probability of false alarm. Interested readers are referred to Van Trees exhaustive work on detection theory for further reading [36].
2.2.5 Example

A sonar data segment is processed using the processing chain described in the previous sections. Fig. 1 shows processed data on each level. Only data from a single hydrophone is shown, but data from all hydrophones are used in the processing. Two threshold-crossings are detected. The first detection is obvious at all processing stages. The second detection is not discernable in the hydrophone and beamformed data and only barely discernable in the matched filtered data. This illustrates the importance of the normaliser as a trend-remover. Notice also how the matched filter effectively pinpoints the location of the first detection.
2.3 Raytracing and sonar performance modelling

Numerical models may be used to estimate sound propagation. Jensen et al. [37] give a thorough description of the most popular modelling methods available for estimating sound propagation in water. One such modelling method is the raytracer, which is a geometrical method that traces rays perpendicular to the wavefront of an acoustic wave. Ray theory was originally formalised to describe the propagation of light in optics and is an extension of Snell’s law, also called the law of refraction, which forces rays to refract towards lower sound speeds as they propagate through the medium. Ray-tracing is a high-frequency approximation that does not take diffraction into account. The work presented in this thesis involves sonar data with frequencies in the kHz domain where the high-frequency assumption holds. This makes raytracing the preferred acoustic model in this thesis.

The following sections give a brief description of raytracing and how it may be used to model reverberation and sonar performance. Two raytracers used in this thesis, PlaneRay and Lybin, are presented.

2.3.1 Basic ray concepts

The list below contains a description of basic ray concepts:

- Refraction is the bending of rays due to changing sound speed along the path of the ray.

- A turning point is where the ray grazing angle changes sign due to refraction.

- Travel time, at a specific raypoint, is the time it takes the ray to propagate from the source to the specific raypoint.

- Initial angle is the grazing angle of the ray at the source.

- Adjacent rays have adjacent initial angles.

- Ray tube is a volume bounded by three adjacent rays, or in the case of 2d raytracing, an area bounded by two adjacent rays.

- Ray intensity at a certain point depends on the area of the ray tube at that point as well as source power, Jensen et al. [37] give a detailed account on how ray intensity is estimated.
2.3.2 Ray categories

Rays are frequently categorised. The ray reflection and refraction history determines what category a ray belongs to. The list below describes the basic categories:

1. Direct path (DP): No reflections and refractions
2. Bottom bounce (BB): One bottom reflection
3. Surface bounce (SB): One surface reflection
4. Upward refracted (UR): One lower turning point (convex shape of the path)
5. Downward refracted (DR): One upper turning point (concave shape of the path)

Fig. 2 illustrates the different categories. Higher order categories include several combinations of reflections and refractions, e.g. BB–SB–BB, a category containing rays reflected off the bottom, then the surface, and finally the bottom again.

2.3.3 Eigenrays

Eigenrays are rays of different categories that propagate from a given source position to a specified target position, see Fig. 2. Jensen et al. [37] suggest different eigenray search methods. One of them, the interpolation method, traces a fan of rays from the source and registers the two adjacent rays of each category that pass each side of the target. The properties of the rays, such as initial angle, travel time, and intensity are then interpolated to find a single eigenray for each ray category.

2.3.4 Transmission loss modelling

Raytracers are frequently categorized as either coherent and incoherent [37]. This relates to how the raytracer estimates the acoustic pressure, \( p(r, z, \theta) \), at a single point. The acoustic pressure is here represented in cylindrical coordinates where \( r \) is the range, \( z \) is the depth, and \( \theta \) is the horisontal angle. Coherent raytracers sum the pressure contribution from each eigenray
crossing the relevant point coherently, that is, a complex summation that includes pressure phase and magnitude:

\[
p^{(c)}(r, z, \theta) = \sum_{j=0}^{N-1} p_j(r, z, \theta),
\]

where \( N \) is the number of eigenrays crossing \((r, z, \theta)\) and \( p_j(r, z, \theta) \) is the complex pressure of eigenray number \( j \). Incoherent raytracers sum the squared
The pressure magnitude of each ray crossing the relevant point:

\[ p^{(i)}(r, z, \theta) = \sqrt{\sum_{j=0}^{N-1} |p_j(r, z, \theta)|^2}. \]  

(7)

The transmission loss, \( tl \), is found using the following expression [37]:

\[ tl(r, z, \theta) = \frac{I(r, z, \theta)}{I_0}. \]  

(8)

The logarithmic expression is more frequently used in literature:

\[ TL(r, z, \theta) = 10 \log_{10} tl(r, z, \theta). \]  

(9)

\( I(r, z, \theta) \) is the intensity:

\[ I(r, z, \theta) = \frac{p(r, z, \theta)^2}{2\rho(r, z, \theta)c(r, z, \theta)}, \]  

(10)

where \( \rho \) is the density, \( c \) is the sound speed, and \( I_0 \) is the intensity at 1 m distance from a reference spherical, free source. The transmission loss is incoherent if the input pressure is estimated as shown in (7), and coherent if the pressure from (6) is input.

2.3.5 Reverberation modelling

The principle behind active sonars is to transmit an acoustic signal from a source and then receive returns on a receiver. Acoustic returns consist of both desired and undesired returns. Desired returns are echoes from targets, e.g. submarines in anti-submarine warfare. Undesired returns, also called scattering, consist of returns from acoustic scatterers in the ocean. The summed contribution of all these scatterers is called reverberation. Reverberation is typically divided into three categories [34]: surface reverberation, volume reverberation, and bottom reverberation. In littoral sonar operations, and particularly for variable depth sonars such as the active, low–frequency, towed array sonar, bottom reverberation typically limits the sonar conditions and causes increased false alarm rates and possibly sonar clutter, as discussed later. Bottom reverberation, \( rl(r, \theta) \), from a scattering patch with area \( A \), may be modelled using the following expression [39]:

\[ rl(r, \theta) = \sum_{j=0}^{N-1} \sum_{k=0}^{N-1} tl_j(r, z_b, \theta)tl_k(r, z_b, \theta)A\sigma(\phi_j, \phi_k), \]  

(11)
where $tl_j(r, z, \theta)$ is the linear transmission loss of a single eigenray from the source to the scattering patch, $tl_k(r, z, \theta)$ is the linear transmission loss of a single eigenray from the scattering patch to the receiver, and $\phi_j$ and $\phi_k$ are the grazing angles of the incoming and outgoing eigenrays at the scattering patch. $\sigma(\phi_j, \phi_k)$ is the scattering function [38]. A commonly used scattering function is Lamberts law:

$$\sigma(\phi_j, \phi_k) = \mu \sin \phi_j \sin \phi_k.$$  

More realistic scattering functions, such as the perturbation model and the Kirchhoff model are described by Hovem [39].

2.3.6 Sonar performance modelling

Sonar performance may be modelled by using the logarithmic sonar equation [34]:

$$SE = SL - TL_f - TL_b - RNL + TS - DT,$$

where $SE$ is the signal excess, $TL_f$ is the transmission loss from the source to the target, $TL_b$ is the transmission loss from the target to the receiver, $RNL$ is the reverberation and noise level, $TS$ is the target strength, and $DT$ is the detection threshold. A thorough description of each parameter can be found in Urick [34]. Transmission loss and reverberation may be estimated using an acoustic model, e. g. a raytracer. The probability of detecting a target at a location where the modelled signal excess level is 0 dB, is 50 %. Receiver operating curves [34] may be used to determine the probability of detection for other signal excess levels. The probability of false alarm depends on the selected detection threshold, see section 2.2.4.

2.3.7 PlaneRay

PlaneRay [40] was developed and is maintained by professor Jens Hovem at Sintef and the Norwegian University of Science and Technology. PlaneRay is a coherent raytracer that uses the interpolation method for determining eigenrays. The raytracer was originally tailored for fast geoacoustic inversion. The idea is that since the water-column raytracing is independent of the bottom properties, PlaneRay is run a single time only during the inversion process. The losses due to bottom reflections are updated for each bottom interacting ray as the bottom properties change during the inversion.

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In this thesis, PlaneRay is used for modelling travel times and initial angles of eigenrays. The eigenrays shown in Fig. 2 are estimated using PlaneRay. In the figure basic eigenrays are presented only, but PlaneRay is also capable of estimating more complex arrivals.

2.3.8 Lybin

Lybin [41] is the property of the Norwegian Navy and was developed by Svein Mjølsnes at the Norwegian Defence Logistic Organisation and is currently maintained by the Norwegian Defence Research Establishment. The model was originally developed for use on sonar vessels in the Norwegian Navy for sonar performance modelling.

In this thesis, Lybin is used for incoherent modelling of reverberation and sonar performance. Fig. 3 shows the graphical user interface of Lybin as well as plots of modelled rays, transmission loss, and reverberation. Lybin has very low computation time, less than 0.1 s for simple cases such as the one shown in Fig. 3.

2.4 Empirical orthogonal functions

Empirical orthogonal functions (EOF) or principal component analysis (PCA) is used for statistical analysis of spatial or temporal variability of physical fields. Preisendorfer and Mobley [42] give a detailed account of PCA techniques on oceanographic and meteorological data. Characterisation of oceanographic data by EOFs was introduced in the 1970s [42], and extended to represent sound speed profiles (SSP) by Tolstoy et al [43] in the early 1990s. The main advantage of characterising SSPs by EOFs is that the information from entire SSPs may be contained in a few scalar coefficients. For example, two EOFs and their coefficients give a sufficient representation of the Munk–profile [23].

EOFs for a set of SSPs may be derived by first interpolating the SSPs to a uniform grid of $N$ depths and then organising them in a data matrix, $C$:

$$
C = \begin{bmatrix}
c_1^T \\
c_2^T \\
\vdots \\
c_M^T
\end{bmatrix},
$$

(14)
where \( c_j \) is an interpolated SSP. Let \( \bar{c} \) be the depth–averaged sound speed profile:

\[
\bar{c} = \frac{1}{M} \sum_{j=1}^{M} c_j. \tag{15}
\]

A single SSP may then be expanded in any set of orthonormal basis vectors [35]:

\[
c_j = \bar{c} + U^T \kappa^{(j)}. \tag{16}
\]

The elements of the coefficient vector \( \kappa^{(j)} \) may be determined as follows [35]:

\[
\kappa^{(j)} = U x_j, \tag{17}
\]

where \( U \) is a matrix containing the transposed basis vectors, \( u_k^T \), on each row, and \( x_j \) is given by:

\[
x_j = c_j - \bar{c}. \tag{18}
\]
The basis vectors are called EOFs. Since the EOFs are orthonormal, then:

\[ u_k^T u_l = \begin{cases} 
1, & k = 1 \\
0, & k \neq 1 
\end{cases} \]  \tag{19}

The EOFs are found by solving the following eigenvalue problem:

\[ R_x u_k = \lambda_k u_k, \]  \tag{20}

where \( \lambda_k \) is the eigenvalue corresponding to the \( k \)th EOF and \( R_x \) is the covariance matrix [35]:

\[ R_x = \frac{1}{M} X^T X, \]  \tag{21}

where the mean subtracted data matrix, \( X \), is given by:

\[ X = \begin{bmatrix} 
x_1^T \\
x_2^T \\
\vdots \\
x_M^T 
\end{bmatrix}. \]  \tag{22}

The method is only meaningful if there is some correlation between the inputted SSPs. Poorly correlated data sets require more EOFs than well correlated data sets for proper representation of the data. The proportion of variances, \( \Lambda_l \), is a useful indicator of this correlation and is frequently used for determining how many EOFs are required for a sufficient representation of the data set:

\[ \Lambda_l = \sum_{k=0}^{l} \frac{\lambda_k}{\sum_{k=0}^{N-1} \lambda_k}. \]  \tag{23}

Note that the eigenvalues are here sorted from largest at \( k = 0 \) to smallest at \( k = N - 1 \). Typically, a threshold \( T \) is selected, and the number of EOFs used is determined as follows:

\[ \min_l (\Lambda_l \geq T). \]  \tag{24}

New sound speed profiles with the same statistical properties as the original data set may be constructed using:

\[ c = \bar{c} + U^T \kappa. \]  \tag{25}
The elements, $\kappa_k$, of the coefficient vector are modelled as zero-mean random processes with variances given by $\lambda_k$ [35]. The probability density function used to model the random process should be selected with care so as to retain the higher order moments as well as the mean and variance.

2.4.1 Example

In connection to the sea acceptance tests of the new Norwegian frigates, a set of 20 SSPs were measured in the Norwegian trench in September 2008, see Fig. 4 (a). Fig. 4 (b) shows the first four EOFs estimated from the measured SSPs. The corresponding eigenvalues are shown in Fig. 4 (c). Observe how quickly the eigenvalues fall off for increasing coefficient number. Using a threshold, $T$, of 0.9, see (24), then according to the proportion of variances shown in Fig. 4 (d) four EOFs are sufficient for representing the measured SSPs. A set of constructed SSPs using (25) are shown in Fig. 4 (e). Gaussian probability density functions are used to model the coefficients. Notice the maxima (at approximately 40 m) and minima (at approximately 60 m) in the constructed SSPs. These extreme values are not observed in the original data set and are probably unphysical. They are generated due to outliers in the first and third coefficient, see Fig. 4 (b). This is a good example of how non-physical artifacts are generated in statistically modelled SSPs when using a random generator that does not model the physics. Such artifacts may be avoided by replacing the Gaussian random generator by a more physical random generator, or by introducing some kind of reality filter. A simple and robust filter here is selecting a maximum and minimum allowed sound speed at all depths. Fig. 4 (f) shows a set of generated SSPs where $c \in [1480 \text{ m/s}, 1512 \text{ m/s}]$. The new set generated SSPs compare better visually with the original set of SSPs.

2.5 False alarm rates in high reverberant conditions

Sea trials in littoral environments with high reverberant conditions show that high-resolution sonars generate particularly many false alarms in presence of ship wrecks and terrain features such as seamounts and underwater ridges [1–4]. Possible causes for the high false alarm rates include false alarm rate inflation [5–8] and non-Rayleigh reverberation [1, 9–16].

False alarm rate inflation is a signal-processing-induced phenomenon that occurs when the reverberation power level is non-stationary in the normaliser
Figure 4: (a) 20 SSPs measured in the Norwegian Trench in September 2008. (b) First (blue), second (green), third (red), and fourth (cyan) EOF derived from the 20 SSP measurements. (c) Eigenvalues corresponding to the first 15 EOFs. (d) Proportion of variances plot for the first 15 EOFs. (e) Original SSPs (red) and constructed SSPs (blue) using (25) with coefficients modelled as random processes. (f) Original SSPs (red) and constructed SSPs (blue) using (25) with coefficients modelled as random processes, but with requirements on maximum and minimum sound speed.
window. E. g., the reverberation in the analysed sample originates from a sea mount, while most of the normaliser window falls to the side of the seamount, resulting in an underestimated background power estimate and therefore increased false alarm rate. This phenomenon is closely related to target masking [5], which occurs in the opposite situation when the seamount is located within the normalisation window resulting in an overestimation of the background level and therefore lost detections.

Non-Rayleigh reverberation is often referred to as sonar clutter. The Rayleigh probability density function is often assumed to model reverberation induced matched filter (MF) envelope well. The reverberation is non-Rayleigh when this assumption does not hold. Use of high-resolution sonars in littoral environments often result in non-Rayleigh distribution of MF data, typically with heavier tailed distributions [1, 9–11, 13–16]. This results in greater false alarm rates than anticipated when assuming Rayleigh reverberation.

Prediction and reduction of clutter and raised false alarm rates are the focus of many published studies. Studies cover different fields such as normalisation [6–8, 17], detection theory [44], image processing [18–20], acoustic modelling [4, 21, 22, 45], and other signal processing techniques [3]. Recently, several papers have been published on which environmental and sonar characteristics control clutter, such as sonar beamwidth [15] and multi-path environments [13, 16]. These papers give useful and insightful descriptions of the clutter phenomenon.

2.6 Acoustic inversion

Bucker [46] introduced matched field processing (MFP), a widely used technique for estimating the location of detected noise sources. Given a known environment, the acoustic fields received on an array of hydrophones from various target locations are modelled. The modelled acoustic field is then compared to the recorded acoustic field by using a cost function, and the target location is estimated by finding the location with optimal cost and therefore best comparison between model and recordings. Detailed descriptions of MFP cost functions and their advantages and disadvantages are found in [47]. Baggeroer et al. [48] give an excellent overview of the work done on MFP up until 1993.

The method has since been extended by including nuisance parameters in the search. Nuisance parameters may for instance be environmental param-
eters or the geometry of the receiver array. These parameters are uncertain and this uncertainty must be accounted for to secure good target location estimates. Focalisation [23] and marginalisation [49] are commonly used methods for including nuisance parameters in the inversion process. Focalisation selects a set of parameters that minimises the cost function, while marginalisation integrates the cost function over all environmental parameters before determining the optimal target location (or whatever other parameters are attempted inverted). Dosso and Wilmut [50] compare marginalisation to focalisation.

Bayesian inversion [51–53] is an extension to MFP. Unlike traditional MFP, Bayesian inversion outputs the posterior probability density function (PPD) for each parameter in the inversion process. The PPD may be used to determine standard deviations, mean estimates, and MAP-estimates for each inverted parameter. This additional information gives an understanding of the quality of inverted parameters.

In the above mentioned work, data from passive sonars were used. MFP has also successfully been extended to low–frequency, active sonars [54] showing promising results for target depth estimation. The main advantage of active sonars is that the propagation time from the sonar to the target is known. The propagation time may be used to estimate the range of the target.

Target location estimation using back propagation was introduced by Tappert [57], and later demonstrated for passive sonars using raytracing [23, 55, 56]. Vertical angles of incoming signals from a target are measured on a receiver array. Rays are traced from the sonar in the direction given by the measured angles. Locations where traced rays intersect are candidate target positions. Back propagation is related to the method called time–reversal [58]. Time–reversal or phase–conjugation is a method borrowed from the field of optics where the received field is retransmitted from the receiver and thereby, due to reciprocity, focused on the source. This method is popular in current underwater communication research. Instead of retransmitting the received field, the back propagation method inputs the environment and the received field into an acoustic model in order to find the target location. The method has recently also been used in air acoustics [59,60]. To the authors knowledge back propagation has not before been demonstrated for active sonar data.
2.7 Acoustic sensitivity to environmental uncertainty

Validity of sonar performance models is generally limited by environmental uncertainty. James and Dowling [29] give an extensive overview of research on how environmental uncertainty influences acoustic field predictions. In littoral environments acoustic propagation is strongly influenced by bottom interaction [25] and the water–column sound speed [24–27]. The acoustic field in littoral environments is therefore very sensitive to uncertainty in the sound speed profile and bottom properties.

The acoustic sensitivity to environmental uncertainty is typically analysed by running an acoustic model repeatedly with Monte Carlo simulated environmental input and then analysing the output using some kind of sensitivity measure, for instance the coefficient of variation [28]:

$$\frac{\sigma(r, z, \phi)}{m(r, z, \phi)},$$

where $\sigma$ and $m$ are the standard deviation and expectation of a suitable acoustic field parameter, e.g. pressure. Both are represented in cylindrical coordinates, $(r, z, \phi)$. The coefficient of variation represents the sensitivity of the acoustic field in a single position, and must therefore be spatially integrated in some way to examine the total sensitivity.

Monte Carlo methods require a large amount of calculations and are therefore time consuming. Other methods have been suggested. Dosso et al. [28] introduce a linearised measure of sensitivity. The underlying assumption is that for sufficiently small environmental perturbation the relationship between changes in the acoustic field is linearly dependent on changes in the environment. If this assumption holds, then the acoustic sensitivity to different environmental parameters may be analysed one parameter at a time, which is far more efficient than a full Monte Carlo approach. Finette [61] shows how environmental uncertainty may be included directly in an acoustic model by incorporating the uncertainty in sound speed in the narrow angle parabolic equation [37].

The uncertainty in sound speed may be reduced by frequent sound speed measurements, but this is costly and not always feasible. An alternative approach is to invert sound speed profiles (SSP) from recorded acoustic data in order to improve the acoustic modelling [23, 52, 56, 62, 63]. Inversion approaches may also be used to obtain other environmental information such as bottom properties. Generally speaking, if the acoustic field is sensitive to
uncertainty in an environmental parameter, then the parameter is eligible for inversion. Inversion techniques using data from naval sonars are frequently called through–the–sensor techniques. One such application is inverting bottom properties from recorded reverberation data [53]. The inverted bottom properties are then used in acoustic modelling in order to improve the results from sonar performance modelling for that system.

3 Description of attached papers

The following subsections give brief summaries of the contents of the papers. All papers are related to at least one of the two topics listed in section 1.2. The papers are listed below together with their status (published or unpublished), my role in writing them, and what topic is addressed:

1. Sonar false alarm reduction using detailed bathymetry data and acoustic propagation modelling
   - Published in the proceedings of Underwater Defense Technology Conference Exhibition 2005 in Amsterdam.
   - Main author
   - Adresses topic A

2. Predicting sonar false alarm rate inflation using acoustic modelling and a high-resolution terrain model
   - Published in IEEE Journal of Oceanic Engineering april 2010
   - Sole author
   - Adresses topic A

3. Target depth estimation using a ray backpropagation scheme on sonar data – simulations and experiments
   - Unpublished manuscript. Submitted to IEEE Journal of Oceanic Engineering
   - Sole author
   - Adresses topic A
4. **Target depth estimation using a ray backpropagation scheme on mid-frequency, active sonar data**
   - Published in proceedings of European Conference in Underwater Acoustics 2010.
   - Sole author
   - Adresses topic A and B

5. **Inverting the water–column sound speed**
   - Published in proceedings of European Conference in Underwater Acoustics 2010.
   - Sole author
   - Adresses topic B

6. **Finding acoustically stable areas through EOF classification**
   - Coauthor
   - Adresses topic B

7. **In ocean evaluation of low frequency active sonar systems**
   - Published in proceedings of Acoustics08 in 2008.
   - Main author
   - Adresses topic B

### 3.1 Summary of the first paper: Sonar false alarm reduction using detailed bathymetry data and acoustic propagation modelling

Active low-frequency towed array sonar systems used in littoral waters experience high false alarm rate, mainly due to reflections from bottom features and steep slopes. This was confirmed by results from two sea trials (2001 and 2002) carried out in the NAT III (New Array Technology) project. The partners in NAT III were TNO-FEL (Netherlands Organisation for Applied
Scientific Research, Physics and Electronics Laboratory), TUS (Thales Underwater Systems) and FFI (Norwegian Defence Research Establishment).

The paper describes an automatic algorithm for classifying detections as false alarms due to raised reverberation levels. Given a high–resolution topography map, the acoustic model Lybin [41] is used to predict received reverberation levels on a specified active sonar. Zones of high probability of false alarms are introduced as geographical areas where normalised modelled reverberation exceeds a preset threshold. Echoes are automatically correlated to predicted zones of high probability of false alarms. Each echo is assigned a value called the percentile overlap. High percentile overlap values indicate that there is a high probability of that echo being generated due to raised reverberation levels.

The method is employed on a data set from the 2002 NAT III sea trial carried out in the Norwegian trench. Data from a small area (8 km by 7 km) with strong variations in the topography was focused on. 5% of the area was predicted as zones of high probability of false alarm, and 60% of the echoes accumulated over 20 pings in the selected area were located within zones of high probability of false alarms.

3.2 Summary of the second paper: Predicting sonar false alarm rate inflation using acoustic modelling and a high-resolution terrain model

This paper refines the method presented in the first paper, and gives a theoretical foundation for why high normalised modelled reverberation levels coincide with increased false alarm rates.

By combining a fast and accurate acoustic model with a high-resolution terrain model, occurrence of false alarm rate inflation may be predicted. The described method outputs the modelled probability of false alarm, which is the probability that a false alarm is generated at a given location due to false alarm rate inflation.

Given high–resolution topography and a measured sound speed profile, the acoustic model Lybin [41] is used to model reverberation for a specified sonar. The modelled reverberation is then normalised using a normaliser window equivalent to the normaliser window used on recorded sonar data from the specified sonar. For a given model–resolution cell, the probability that a single sample in matched filtered and beam formed sonar data exceeds
the detector threshold is derived from the normalised modelled reverberation. This probability is the probability of false alarm due to false alarm rate inflation.

The presented method is used on a data set from the 2002 NAT III sea trial in the Norwegian trench. Modelled probability of false alarm is shown to compare well with spatial concentrations of recorded false alarms, but the modelling underestimates the probability of false alarm by approximately a factor of four. The cause of the underestimation is probably that the method does not completely predict the true probability of false alarm, since other causes of false alarms may also be present, e.g., clutter and noise spikes.

3.3 Summary of the third paper: Target depth estimation using a ray backpropagation scheme on sonar data – simulations and experiments

This paper presents a method that uses ray backpropagation, see section 2.6, on active sonar data in order to estimate the depth of a detected target. A narrow vertical beamwidth of the sonar is required, since accurate vertical arrival angle measurements are needed. The arrival time, vertical arrival angle, and measured environment are inputted in the acoustic raytracer PlaneRay [40]. Due to uncertainties in measured arrival angle a fan of rays are traced with initial vertical angles within two standard deviations of the measured vertical arrival angle. Each ray is assigned a probability determined from a Gaussian probability density distribution with an expectation given by the measured vertical angle. The probability of the target being located at a specific depth is determined by summing the probability contribution of all rays with end points at that depth. The target depth probability density function is determined, and a maximum a posteriori (MAP) estimate and an estimate for the standard deviation are extracted. This information may be used for classification, for instance by prioritising targets with target depth MAP estimates above the sea floor. The estimated standard deviation is a measure of how reliable the target depth estimate is.

The fidelity of the method is studied by simulating different environments with detected targets at different depths and ranges and with different signal-to-reverberation and noise ratios (SNR). The target depth estimates are shown to deteriorate for increasing ranges and decreasing SNRs. In simple scenarios and for sufficiently high SNR (23 dB), the method is shown
applicable for ranges up to 15 km.

The method is finally tested on recorded data for targets located on the sea floor and in the upper half of the water column. The method successfully estimates the target depth with an accuracy sufficient for classification purposes 82% of the time. Averaged over all measurements the target depths are estimated within 40 m of the true depth.

3.4 Summary of the fourth paper: Target depth estimation using a ray backpropagation scheme on mid-frequency, active sonar data

In this paper the method described in the third paper is developed further and given a stronger theoretical foundation. As in the third paper ray backpropagation is used, but is here combined with a Bayesian inversion approach [51] to estimate the a posteriori target depth probability density function. The introduction of Bayesian theory allows inclusion of a priori probabilities assigned to the environment. Tuning of the environment, by means of focalisation [23], is included in order to improve target depth estimation. Two environmental parameters are considered; sonar depth and sound speed profile. The former is included to take into account ship heave due to surface waves and ship motion. The latter is included by using empirical orthogonal functions (EOF) to represent sound speed profiles. Two EOFs are assumed sufficient for describing the sound speed variability. The sonar depth and EOF coefficients are varied in order to find the MAP estimate of the target depth. The computation time of this method far exceeds the computation time of the method described in the third paper, but the inclusion of focalisation improves the accuracy of the results.

3.5 Summary of the fifth paper: Inverting the water–column sound speed

This work presents an inversion method for estimating sound speed profiles by exploiting available sensor information. Sensor information considered includes sound speed measurements close to sonar equipment and echo sounder data used to estimate depth–averaged slowness. The depth–averaged slowness may be estimated from echo sounder data if the bottom depth is known.

Empirical orthogonal functions are determined from a set of known sound
speed profiles. The sound speed profiles used may be climatological data or, as used in this work, modelled sound speed profiles, e.g. from the MIPOM ocean model [64]. Sound speed profiles are generated by varying the weights of each EOF. Analytical and differentiable expressions, that include the EOF–coefficients as the only variables, are derived for each type of measurement (direct sound speed measurements and depth–averaged slowness). A conjugate gradient search is used in order to estimate a sound speed profile that matches well with measured values collected from the sensors.

A common problem during sonar operations is to determine how often to measure the sound speed profile. An algorithm for assessing the quality of the most recently measured sound speed profile is introduced. The sonar performance modelled using the most recent sound speed profile measurement is compared to the sonar performance modelled using an inverted sound speed profile. The hypothesis is that when the most recent measured sound speed profile results in a poor comparison, then a new sound speed profile should be measured. This is equivalent to a binary decision problem. The algorithm was applied on a simulated data set, and the probability of detection (the probability of deciding that a poor–quality measured sound speed profile is of poor quality) and probability of false alarm (the probability of deciding that a high–quality measurement is of poor quality) were shown to be 61% and 6%, respectively.

For a simulated data set, sonar performance predictions based on inverted sound speed profiles were shown to be comparable to performance modelled on basis of four-hourly sound speed measurements. This indicates that for simple sonar performance modelling, for instance for modelling the expected detection range during sonar operation, the presented inversion method may be used instead of measurements of the sound speed profile.

3.6 Summary of the sixth paper: Finding acoustically stable areas through EOF classification

Validity of sonar performance models is generally limited by environmental uncertainty [29], and particularly uncertainty in the sound speed profile (SSP) [24–27, 65]. Rapid environmental assessment (REA) missions, e.g. using gliders, and advanced ocean models may be used to reduce this uncertainty prior to sonar operation.

This paper presents a method on how EOFs may be used for locating
acoustically stable water masses in otherwise unstable waters. Acoustically stable water masses are defined as areas where modelled target signal excess has low sensitivity to expected oceanographic variability. A simple stability measure based on modelled signal excess is derived in order to measure the acoustic stability of an area.

A map of acoustically stable areas is the main output. This output is for instance useful for planning deployment of gliders during a REA mission. Large, geographically contiguous groups indicate acoustically stable areas where frequent SSP measurements are unnecessary, e.g. low concentration of gliders. Geographically mixed groups indicate the opposite. Other applications include determination of suitable locations for sonar tests that require stable sonar conditions and efficient optimization of sonar parameters in acoustically stable areas.

Modelled oceanography from the MI-POM ocean model [64] for an area close to the Western coast of Norway is used as an example. Surface salinity is a commonly used indicator for classifying water masses as either Atlantic water or coastal water. A simple comparison of the distribution of the first EOF coefficient and the surface salinity values shows that EOFs are also useful for classifying water masses. Based on the modelled sound speed profiles, the area is divided into acoustically stable subareas using the method described above. Both large contiguous groups and smaller, geographically mixed groups are generated. The locations of the geographically mixed groups match well with areas where mixing of coastal water masses and Atlantic water masses supposedly occurs, while the larger groups coincide well with homogeneous water masses.

3.7 Summary of the seventh paper: In ocean evaluation of low frequency active sonar systems

All though this paper does not directly address the topic B, as described in section 1.2, the relevance is strong enough for the work to be included here. Unlike the other papers, the work presented in this paper relates to a non-operational scenario, namely acceptance tests for naval sonars at sea.

Sonar performance measurements in the sea are always affected by uncontrollable and/or uncertain environmental conditions, such as sound speed variations, bottom topography, or the acoustic properties of the sea floor. This paper presents a method to determine a sonar – target geometry which
minimizes the uncertainty in target signal excess due to environmental variability.

An acoustic model is used to estimate signal excess for a large number of sound speed profiles measured in the relevant area. The results are compared while searching for a target range and depth where estimated signal excess is robust with respect to the expected variability of the sound speed profile in the actual area.

The achieved sensitivity of signal excess to environmental changes is demonstrated for different test geometries. Robustness in signal excess is shown to be highly dependent on target range and depth and sonar depth. Careful selection of the sonar – target geometry may reduce the uncertainty in modelled signal excess.

4 Conclusion

The thesis contains seven papers that address two relevant topics of research:

A How to exploit available environmental information in order to increase the classification ability of anti–submarine warfare (ASW) sonars

B How to deal with environmental uncertainty

The first topic is addressed by developing new classification algorithms. The first two papers present methods of predicting what areas are prone to high false alarm rates. The predictions are based on detailed environmental knowledge and acoustic modelling. The third and fourth papers present methods were vertical beamforming of sonar data is exploited in order to find the vertical arrival angle of target echoes. Ray backpropagation is then used to estimate target depth. Target depth is a very useful classification clue, and the method is proved sufficiently accurate for classification.

The achilles heel of the proposed classification algorithms is their need for accurate environmental information. Uncertainty in sound speed profile may result in ambiguous or erroneous results. The second topic deals with methods that reduces the uncertainty in the sound speed profile. The fourth and fifth papers present methods on how the sound speed profile may be extracted from data recorded during sonar operation. The last two papers present methods that analyse the acoustic stability of geographical areas. The assessment is based on a large set of sound speed profiles from the
analysed area. This data set can either be obtained from an ocean model or be densely measured sound speed profiles.

Papers six and seven introduce methods that are useful for determining acoustically stable areas for conducting sea trials in, for instance sonar tests. During the sea acceptance tests for the sonars on the new Norwegian F310-class frigates, these methods have been employed successfully. The oceanographic field was sampled densely using a moving vessel profiler. The method described in the sixth paper was then used to find acoustically stable areas within the measured area. On basis of the sound speed profiles measured in the selected area, the method in the seventh paper was then used to find the optimal sonar–target geometry for the acceptance tests. By optimal is here meant minimised uncertainty in the acoustic field.

4.1 Future work

Listed below are unresolved issues that are suggested for future work.

4.1.1 Normalisation optimiser

The second attached paper introduces a method where reverberation modelling is used to find areas susceptible to false alarm rate inflation. The occurrence of false alarm rate inflation depends not only on the present environment, but also on the sonar parameters used and particularly the normaliser used. The developed method may be extended to automatically configure the normaliser in order to reduce false alarm rate inflation, e.g. by varying the normaliser window and guardband sizes.

4.1.2 Countering target masking

Another possible extension of the method introduced in the second paper is prediction of target masking. Target masking is often exploited tactically in order to hide from radars, for instance by placing military assets next to forests or other strong scatterers. Likewise, in naval warfare, submarines could hide in front of strong upslopes or seamounts to avoid detection. This tactic can be countered by predicting what areas are prone to target masking and then use one–sided normaliser windows to avoid the effects of target masking.
4.1.3 Automatic classification on basis of target depth estimation

The target depth estimator introduced in the third and fourth papers is suitable for implementation in combat management systems as a classification tool. The next step should be to test a prototype version live during sonar operation. An unresolved issue is how to best exploit multiple ping information. The two papers introduce different ways of coping with this problem. The method used in the fourth paper is most refined but also very slow. The computational cost could be reduced by implementing an improved search algorithm, such as simulated annealing. Another problem is that the assumption of independence between pings made in equation (13) in the fourth paper is in some cases questionable. The arrival time and angle measurements are probably independent, but the environmental input is not. If the assumption of independence is invalid then other means of exploiting multiple ping information must be made, such as the ones described in the third paper.

4.1.4 Sound speed profile inversion

The fifth paper presents a method for inverting the sound speed profile from echo sounder data and direct sound speed measurements. This method has been tested on a simulated scenario only and should therefore be tested on measurements for verification. The next step would be to make a prototype version live on a sonar vessel. The inverted sound speed profile can then be used either to check if the most recently measured sound speed profile is valid or to be used as input to sonar performance modelling. This could also be combined with the target depth estimator or normalisation optimiser to improve the results.

4.1.5 Spatio–temporal assessment of acoustic stability

The method for categorising the acoustic stability of geographical areas presented in the sixth paper may be extended to take into account temporal variability. By analysing spatio–temporal variations, it should be possible to estimate how often sound speed measurements should be made to ensure a proper sampling of the environment during for instance sonar operation or rapid environmental assessment missions.
4.1.6 Acoustic sensitivity analyses for different sources of environmental uncertainty

The method described in the seventh paper has been successfully put to use during the sea acceptance tests for the Norwegian F310 frigates. However, the uncertainty in the sound speed profile is the only environmental uncertainty considered. The method may easily be extended to include other environmental parameters such as wind speed, bottom depths, and bottom properties. Furthermore, geometric uncertainties such as uncertainty in sonar depth and target location may also be included. Assuming locally linear acoustic sensitivity to each of these uncertain parameters, then stability plots that combine uncertainty in all these parameters simultaneously may easily be made. The contribution of each parameter may also be studied separately in order to determine what parameters the acoustic field is most sensitive to uncertainties in. This may shed light on how a test procedure may be improved to reduce the uncertainty of the test results.

Another possible extension is to find more objective ways of determining whether a certain situation is acoustically stable. The current method is subjective since it requires visual inspection of stability plots to determine the stability.

References


Paper 1

Sonar false alarm reduction using detailed bathymetry data and acoustic propagation modelling
Sonar false alarm reduction using detailed bathymetry data and acoustic propagation modelling

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1 Abstract

Active low-frequency towed array sonar systems used in littoral waters experience high false alarm rate, mainly due to reflections from bottom features and steep slopes. This was confirmed by results from two sea trials (2001 and 2002) carried out in the NAT III (New Array Technology) project. The partners in NAT III were TNO-FEL (Netherlands Organisation for Applied Scientific Research, Physics and Electronics Laboratory), TUS (Thales Underwater Systems) and FFI (Norwegian Defence Research Establishment).

This paper describes a method of reducing the false alarm rate using an acoustic model. Predicted zones of high probability of false alarms are introduced as geographical areas where the normalised modelled reverberation exceeds a preset threshold. A method of automatically correlating recorded monostatic echoes with the predicted zones is presented. A probability of false alarm is linked to the echoes for tracking purposes, resulting in lower track probability in zones of high probability of false alarms.

2 Introduction

This paper is based on work in the NAT III programme. NAT III, was a co-operation between TNO-FEL, TUS and FFI. The purpose of the programme was to assess the advantages of bistatic operations with low frequency active sonars, LFAS. An LFAS system was tested in Norwegian waters in two sea trials in 2001 and 2002. In particular, the system’s performance in shallow and coastal waters was evaluated. The programme was closed in November 2004. The data presented in this paper is from the 2002 sea trial.

The main advantages of LFAS systems are the high beam resolution and the good performance at long distances. A problem in fjords and in shallow, coastal waters is the amount of echoes2 generated from each transmission3. Submarines are not the only reflectors, but terrain features causes cluttering of echoes. These false echoes behave similarly to submarine echoes, and they are in great numbers. An ideal tracker4 is easily jammed if fed by many echoes, and is not able to process the data in real-time. Approaches to deal with this problem can roughly be divided into two categories; simplifying the tracker or reducing the amount of echoes. The method suggested in this paper is of the latter sort. The idea is to use knowledge on the

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2 Echoes are received reflections of the transmitted pulse from reflectors in the sonified medium.
3 A transmission is the acoustic energy transmitted by the sonar.
4 A tracker is an algorithm that creates a path of a hypothetical target using series of echoes from different transmissions.
environment in combination with an acoustic propagation model in order to predict what areas are most likely to generate false echoes. This information can be used to either remove recorded echoes within these areas or reduce the track probability of tracks generated in them. Either solution would reduce the computational cost of the tracking process, allowing more advanced and accurate trackers to be used.

The method described is based on the work presented by Jon Wegge at the Underwater Defence Technology conference in Malmo 2003, see ref [1].

3 Low frequency active towed array sonars in Norwegian waters

Norwegian waters offer a wide range of environmental challenges due to the complexity in the oceanography and variations in the sea floor terrain. The north Atlantic Gulf steam interacts with coastal streams and fresh water from the land. The terrain varies between archipelago, deep fjords, shelves, and deep ocean conditions. In addition the waters are rich in terms of biology, not only introducing high false alarm likelihood, but also imposing restrictions on the use of LFAS.

Detailed hydrographic mapping enables us to better model the sonar performance. This again makes us more capable of predicting the likely locations of echoes from bottom reverberation, in addition to the range and path of acoustic energy.

Sonar processing in the past did not add detailed information neither from the terrain nor from any acoustic model. It was basically left to the operator to interpret the sonar response. However, with new sonars, the false alarms increase in number as a result of the increased range and bandwidth, but the alarms may be more accurately localized as a result of a narrower beam width. Thus there is a need for a more effective false alarm reduction filter, a method that is closer to reality as a result of the more accurate echo localization and detailed terrain information.

4 Method of correlating echoes with terrain

This section describes the method used to automatically correlate echoes with modelled zones of high reverberation. The first subsection presents the acoustic raytrace model, LYBIN. The second subsection defines zones of high probability of false alarm, and describes the method of predicting them. The third section describes how recorded echoes are correlated with zones of high probability of false alarm.

4.1 Acoustic model and reverberation modelling

The acoustic propagation model used is LYBIN. LYBIN is an incoherent ray trace model developed by Svein Mjølsnes at NDLO/Sea (Norwegian Defence Logistic Organisation). It models the transmission loss and reverberation in a single vertical cross section and uses the sonar equations, see ref [2], to compute the signal excess and probability of detection of a hypothetical target within the cross section. Input is the sonar parameters\(^5\), a depth dependent sound speed profile, wind speed\(^6\), bottom

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\(^5\) Such as sonar position, source level, frequency band, pulse length, beam widths and side lobe levels.
parameters\textsuperscript{7} and range dependent bottom depths. Recent developments of LYBIN have made it range dependent in wind speed, sound speed and in bottom parameters as well.

A single LYBIN run computes the reverberation in a single direction from the sonar. LYBIN must therefore be run once for every direction of interest. The general problem requires a full, 360\textdegree coverage. In the examples shown in this paper a resolution in direction of two degrees has been used, that is LYBIN is run 360 times for every ping, and the direction of each LYBIN run is separated by 1\textdegree. This method of using a 2d acoustic propagation model in a 3d problem, is in literature referred to as the n*2d method. The range resolution used is 45m. The depth resolution varies. 50 depth cells are used and the maximum depth depends on the current bottom profile.

\textbf{4.2 Predicted zones of high probability of false alarms}

Predicted zones of high probability of false alarms are areas where the acoustic model predicts local maxima in the reverberation. The reverberation is computed using LYBIN and the n*2d method. The reverberation is then normalized in range using:

\[
\text{Norm}(\text{rev}_i) = \frac{\text{rev}_i - \overline{\text{rev}}_i}{\text{std}(\text{rev}_i)} \tag{Equation 1}
\]

\(\text{rev}_i\) is the modelled reverberation in range cell \(i\). \(\text{Norm}(\text{rev}_i)\) is the normalized reverberation in range cell \(i\). \(\overline{\text{rev}}_i\) is the average reverberation in two split-windows to each side of range cell \(i\). The windows have widths of 15 cells (675m), and five cells separate them (225m). \(\text{std}(\text{rev}_i)\) is the estimated standard deviation using the same windows as in the averaging.

After normalization the data is thresholded to find the local maxima. In the example shown in this paper, the threshold is 8dB. (This threshold must not be confused with the threshold commonly used in detectors to extract echoes, see next section.) Note that the model resolution in range, the width of the normalization windows and the threshold value are interconnected. Peaks in the modelled reverberation are typically reduced when the model resolution decreases. The reason is that the reflected energy from bathymetric features is smeared out over large range cells. The size and separation of the normalization windows also directly influence the peaks in the normalized reverberation. Finally, the preset threshold determines which of these peaks result in modelled detections. Figure 1 illustrates the procedure for a single direction. This is done for all \(n\) directions, resulting in a set of discreet positions where the modelled and normalized reverberation exceeds a threshold.

\textsuperscript{6}The wind speed is used to compute the surface back scatter, bubble attenuation close to the surface, ambient noise level and also surface forward scattering.

\textsuperscript{7}LYBIN uses a single-valued parameter between 0 and 10 to classify all bottom types for both bottom backscatter and loss computations.
Figure 1: Left plot is modelled reverberation in dB rel 1 Pa. The right plot is the same data after normalization. The black dashed line in the right-hand plot is the threshold value.

Each range cell in a single LYBIN run represents an area, henceforth called area cells, which is equal to:

$$A = r \Delta r \Delta \theta$$

Equation 2

$r$ is the range. $\Delta r$ is the size of a range cell and equals the maximum model range $R$ divided by the number of range cells. $\Delta \theta$ is the bearing resolution, that is the difference in angle between neighbouring LYBIN runs. The size of the area is proportional to range, which means that larger ranges results in higher uncertainty, and less ability to recognize bathymetric features. Keep in mind that a single bottom profile is used for each direction. This means that a prominent bathymetric feature within an area could be missed by the bottom profile, and therefore not modelled correctly. Higher resolution in bearing solves this problem, but bear in mind that the computational effort is inversely proportional to the bearing resolution, when using the $n*2d$ method.

We realize that the exact position and extent of a clutter is uncertain. To make the method more robust to localisation errors, we extend all predicted areas to including neighbouring cells and we also merge areas that are very close. An imaging technique called dilation is used. Figure 2 illustrates the procedure. The areas are here called modelled detections, or just detections in the figure. The red squares represent detections. Yellow squares represent neighbouring cells. Cyan squares represent common neighbours of two or more modelled detections. A zone of high probability of false alarms is an area consisting of red, yellow and cyan cells in contact, directly or indirectly. In the example there are five zones, each of them are numbered and bounded by a red box.
4.3 Correlating recorded echoes with zones of high probability of false alarms

The correlation of recorded echoes with modelled zones of high probability of false alarms is simple and straightforward. Each recorded echo’s area of uncertainty is geographically compared to the location of zones. If they overlap, then the echo is defined as linked to that zone with a percentile overlap equal to the ratio of the overlapping area and the total uncertainty area of the echo. Figure 3 illustrates the concept. The intention of the percentile overlap (PO) is to use it in the tracking process. High PO of an echo should lower the probability of a track using it.

\[
PO = \frac{dA}{A} \times 100\%
\]

Correlation percentage of a single transmission is a statistical parameter that shows how many echoes that are linked to a zone, compared to the total number of echoes for that transmission.
5 Results from correlation of recorded echoes and zones of high probability of false alarm

An area of 8km by 7km in Norwegian waters is used to illustrate the method. The area has strongly varying depth, ranging from 0m to 600m. Such varying bathymetry typically results in huge amounts of echoes. For the transmissions presented here, the average number of recorded echoes within the area is 2500. No methods of echo-reduction were applied.

Figure 4 shows echoes and zones of high probability of false alarm for four consecutive pings in a 8km by 7km area in littoral waters. The red echoes are linked to a zone, while the blue echoes are not. Clusters of echoes are bounded by ellipses and numbered. Most of these clusters consist of red echoes, especially the large clusters. A cluster with mainly red echoes is assumed predicted by the model. The correlation between clusters of echoes and zones of high probability of false alarm is good. The model does not easily predict lone echoes, but they seldom generate long living tracks anyway. In ping 17 it seems that there is a shift in angle of all echoes relative the sonar, reducing the correlation. This is most likely due to an error in the array heading, such errors influence positioning of echoes directly.

Clusters 12, 14, 16 and 17 are not predicted. Common for these clusters, are that they are generated in deeper terrain than the predicted clusters. Other studies on the same data set indicate that the sound speed profile used in the modelling has a too strong
sound speed channel. This results in entrapment of too much acoustic energy within the channel. See Figure 5. The peak labelled 1 is the probable cause for cluster 12, while the peak labelled 2 is the cause for cluster 11. A weaker sound channel would result in more acoustic energy propagating into the depths, and therefore deeper zones of high probability of false alarm.

![Figure 5: Transmission loss plot towards cluster 12 in ping 19.](image)

Figure 6 shows statistics of how well the echoes and zones of high probability of false alarm are correlated for twenty pings. The average amount of echoes per ping is 2500 for the area studied. For most pings the correlation percentage exceeds 50%. There seems to be a shift in angle of all echoes in pings 2, 17 and 20, due to an error in the array heading. This is easily seen for ping 17 by studying Figure 4.

![Figure 6: Correlation percentage in example 1 for 20 pings. Correlation percentage is the percentage of echoes that are linked to a zone of high probability of false alarm.](image)
6 Thoughts on using the results in a recursive tracking algorithm

A popular and cost-effective algorithm for target tracking in a cluttered environment is the Interacting Multiple Model combined with the Probabilistic Data Association Filter (IMMPDAF) [3]. This filter manages to initiate, maintain and terminate tracks, and yields the probability that there is a target in track. As the algorithm is presented in [3] it ignores the information about the amplitude or the signal to noise ratio (SNR) of the echo. An improvement of the IMMPDAF to include means to benefit from the amplitude information (IMMFDAFAI) is presented in [4]. This made the data association better by weighing the echoes within the validation gate by their SNR, combined with the traditional weighing of echoes due to their position deviation compared to the predicted position. Further it also included using additional models when a track entered the maintenance mode. This allowed for better tracking under the manoeuvring of targets.

Our intention is to decrease the probability of detection ($P_D$) when our Percentile Overlap (PO, see Figure 3) is increasing. Our PO must not be confused with the probability of false alarm ($P_F$) [5]. How PO is related to $P_D$ and SNR is not defined, but some pragmatically statements for the tracking algorithm can be defined:

- a) Echoes with $\text{PO} > \tau$ can not initiate tracks
- b) Tracks in initiation mode (age < 10 pings) [4] can not use echoes with $\text{PO} > \tau$
- c) The amplitude ($a$) of echoes ($M$) with $\text{PO} \geq 0$ within the validation gate [3] will be weighted ($w$) based on a function of PO and $a$: $w_i = f(\text{PO}_i, a_i), \ i \in M$.

Here $\tau \in [0,1]$ is a fixed threshold for PO to allow tracking in areas with heavy density of bottom reverberation.

The computational cost of the algorithm will be reduced by the statements a) and b). With statement c), the weight of the echo with a high PO will be decreased, and thus have a reduced influence on the data association of new measurements.

In some cases, the echoes from bottom reflection gain a higher SNR than the submarine close to the area. This is due to the differences in depths and size of the reflecting areas of the bottom feature and the submarine. When using the target tracker with amplitude information feature, we will suppress the echo from the submarine in favour of the stronger echo from the bottom feature, and possibly lead the tracker off the target. This opens for an investigation on the possibility of estimating the amplitude of a target, when tracking a target with some defined variation in amplitude. For estimation one have to expand the system models to include the SNR as a state variable. For computational cost effectiveness, this can be combined with the use of Percentile Overlap. Doing so, this can be used to define an additional maintenance mode for tracks near areas with high probability of false alarms due to reverberation.

The Percentile Overlap is expected to be an effective input variable to tracking algorithms to reduce false target tracks based on bottom reverberation, and to reduce computational cost of the tracking algorithms.
7 Conclusions
The method presented predicts that 5% of an 8km x 7km area are zones with high probability of false alarms. Comparison with real sonar echoes for that area shows that an average of 60% of the echoes falls within these zones. Each echo within a zone is assigned a Percentile Overlap. The Percentile Overlap can be used to reduce computation cost of tracking algorithms. It can also reduce the probability of tracks initiated based on echoes from bottom features, and possibly help the algorithm track targets through areas with dense clutter from bottom reverberation.

The method is sensitive to errors in array heading and environmental information. Errors in array heading cause a shifting of angle of all echoes. If the echoes are misplaced, then the model has no real chance in linking them to zones of high probability of false alarm. Errors in the sound speed profile cause the sound to propagate along wrong paths, typically displacing the zones of high probability of false alarm to either shallower or deeper areas. Finally, the method demands a high-resolution bottom depth grid in order to model the bottom reverberation with any success.

8 References
Paper 2

Predicting Sonar False Alarm Rate Inflation Using Acoustic Modeling and a High-Resolution Terrain Model
Predicting Sonar False Alarm Rate Inflation Using Acoustic Modeling and a High-Resolution Terrain Model

Karl Thomas Hjelmervik

Abstract—False alarm rates several orders of magnitude higher than the designed false alarm rate are frequently observed on active, low-frequency, towed array sonars. Increased false alarm rate originates from at least two effects: clutter and false alarm rate inflation (FARI). Clutter, or non-Rayleigh probability distributions of the matched-filter (MF) envelope, is often observed on high-resolution sonars, since too few scatterers are resolved for the central limit theorem to hold. FARI is a signal-processing-induced source of false alarms that occurs when the reverberation is nonstationary in the normalizer window. For example, the reverberation in the analyzed sample originates from a seamount, while most of the normalizer window falls to the side of the seamount, resulting in an underestimated background power estimate, and therefore, increased false alarm rate. By combining a fast and accurate acoustic model with a high-resolution terrain model, occurrence of FARI may be predicted. The described method outputs the modeled probability of false alarm, which is the probability that a false alarm is generated at a given location. The method is tested by comparing spatial concentrations of measured false alarms to modeled probability of false alarm. Comparison shows that a significant amount of false alarms is generated due to FARI, and that occurrence of FARI can be predicted given detailed environmental input.

Index Terms—Acoustic modeling, false alarm prediction, false alarm rate inflation (FARI), reverberation, sonar.

I. INTRODUCTION

THE Norwegian Navy is procuring new antisubmarine warfare frigates, the F310-class frigates. The frigates are equipped with active towed array sonars. These are high-resolution sonars with narrow horizontal beamwidths and large-frequency bandwidth relative to the center frequency. The main advantage of high-resolution sonars is their ability to reduce the interference from reverberation since the sonar resolution cells are small and therefore contain relatively few scatterers.

Sea trials in littoral environments show that high-resolution sonars generate particularly many false alarms in the presence of ship wrecks and terrain features such as seamounts and underwater ridges [1]–[4]. False alarms are threshold-crossing peaks in normalized, beamformed, and matched-filtered (MF) data that appear at locations not containing an intended target.

A sonar is typically designed to have a specific false alarm rate [5, Ch. 12.5]. Situations occur where the observed false alarm rate exceeds the designed false alarm rate by several orders of magnitude. There are several possible causes for increased false alarm rates. One is false alarm rate inflation (FARI) [6, Ch. 7.4]–[9]. This is a signal-processing-induced phenomenon that occurs when the reverberation power level is nonstationary in the normalizer window. For instance, in areas with strong topographic variations, the window may be large enough to contain both soft sediment bottoms and steep seamounts with rocky substrate. The latter terrain generates higher reverberation levels than the former, resulting in nonstationary reverberation in the normalizer window. Another important cause for increased false alarm rates is non-Rayleigh-distributed MF envelope [10], [11], often referred to as clutter. Traditionally, the Rayleigh probability density function is used to model reverberation-induced MF envelope to estimate the false alarm rate for a given detector threshold [5, Ch. 8.16]. However, use of high-resolution sonars in littoral environments may, in some cases, result in non-Rayleigh distribution of MF data, typically with heavier tailed distributions [11]–[18]. This results in a greater false alarm rate than designed.

Prediction and reduction of clutter and raised false alarm rates are the focus of many published studies. The studies cover different fields such as normalization [7]–[9], [19], detection [20], image processing [21]–[23], acoustic modeling [2], [3], [24], [25], and other signal processing techniques [1]. Recently, several papers have been published on which environmental and sonar characteristics control clutter, such as sonar beamwidth [16] and multipath environments [15], [17].

In this paper, a new model for predicting FARI is presented. The model combines an accurate and fast acoustic model with detailed knowledge of the environment. Output of the model is the modeled probability of false alarm, which is the probability of a false alarm appearing at a given bearing and range due to FARI. This metric is determined by normalizing the modeled reverberation data with a normalizer equivalent to the one used on the received MF data. Areas with high-normalized, modeled reverberation are prone to FARI, and are henceforth referred to as FARI areas. Derived modeled probability of false alarm, in a single location, depends on the density and strength of FARI areas in the vicinity.

The method is tested on an experimental data set from a sea trial in 2002, and is shown to compare well with areas of
high measured false alarm densities. The sea trial was part of the New Array Technology III project (NAT III), a joint collaboration of the Norwegian Defence Research Establishment (FFI), the Royal Norwegian Navy (RNoN), the Royal Netherlands Navy (RNLN), the French Ministry of Defense (FMoD), the Dutch Ministry of Defense (DMoD), Thales Underwater Systems (TUS), and The Netherlands Organisation for Applied Scientific Research (TNO). The particular data set used in this paper was designed to maximize the amount of false alarms. An active, low-frequency, towed array sonar was towed by the FFI owned R/V H. U. Sverdrup close to shore in an area densely populated by seamounts and steep up-slopes.

The reverberation is modeled using the acoustic raytrace model Lybin [26]. Lybin is the property of the Norwegian Navy, was originally developed by Svein Mjølsnes in the 1980s, and is currently maintained by FFI. Various model components of Lybin have been tested by both NURC [27] and FFI [28].

II. THEORY

This section is split into three subsections. Section II-A gives a brief introduction to normalization. Section II-B discusses how nonstationarity in the received reverberation power may result in FARI and target masking. Section II-C describes a method of how an acoustic model combined with a high-resolution terrain model can predict what areas are susceptible to FARI.

A. Normalization

Signal processing is required for an active sonar to detect a target embedded in reverberation and noise. Typically, the received signal is beamformed and MF. MF data is then normalized before a threshold is applied to find echoes. Echoes consist of intended detections of real targets, such as submarines or torpedoes, but also of unintended detections, so called false alarms. The normalizer, also known as constant false alarm rate (CFAR) processor [6, Ch. 7]–[9], ensures that the false alarm rate remains constant at all ranges. Then, the false alarm rate can be controlled by selecting a suitably high threshold. Detailed accounts on detection theory can be found in [6, Ch. 6].

Let \( \Lambda[i] \) be the beamformed and MF received signal in a single beam in sample number \( i \), then the normalized data \( s[i] \) is given by

\[
s[i] = \frac{\Lambda[i]}{\mu_{\Lambda[i]}}, \tag{1}
\]

\( \mu_{\Lambda[i]} \) is an estimate of the background level at the analyzed sample. Estimation of \( \mu_{\Lambda[i]} \) varies for different normalizers. A commonly used normalizer is the cell-averaging normalizer (CA-CFAR) [6, Ch. 7.2] that estimates background by averaging received MF data in a window of size \( L \) close to the analyzed sample, for instance, by using a split window [7]

\[
\mu_{\Lambda[i]} = \frac{1}{L} \sum_{l=1}^{L/2} \Lambda[i - K - l] + \frac{1}{L} \sum_{l=1}^{L/2} \Lambda[i + K + l] \tag{2}
\]

where \( L \) is the total size of the normalizer windows and \( K \) is the number of samples between the analyzed sample and the windows, often called guard cells in the literature [6, Ch. 7.2], [7]. A more general expression for estimating the background from neighboring samples that applies to most normalizers is

\[
\mu_{\Lambda[i]} = \frac{1}{C} \sum_{l=-L/2}^{L/2} c[l] \Lambda[i - l] \tag{3}
\]

where

\[
C = \sum_{l=-L/2}^{L/2} c[l], \tag{4}
\]

\( c[l] \in [0, 1] \) are coefficients determined by the selected normalizer. For instance, the coefficients in the split window CA-CFAR normalizer described above equal 1 within the normalizer window and 0 otherwise. Note that \( c[0] = 0 \) for most normalizers, since the analyzed sample should not be included in the background estimate.

B. Nonstationary Reverberation in the Normalizer Windows

CA-CFAR normalization relies on stationary reverberation within the normalizer windows with statistics equal to the statistics of the background reverberation of the analyzed sample [8]. Nonstationary reverberation may result in target masking and FARI [7]–[9]. An introduction to these concepts, in a radar context, is found in [6, Ch. 7.4].

Target masking occurs when the average reverberation power is higher in the normalizer window than the true background reverberation power at the analyzed sample. This results in an exaggerated background estimate, and therefore, an underestimated normalized level, which is easily seen from (1). This is called target masking because a target close to, but not inside, a strongly reverberating region may remain undetected due to an underestimated normalized level.

FARI occurs when the background reverberation of the analyzed sample is higher than the average reverberation in the normalizer windows. In this case, the background is underestimated, and the normalized level exaggerated. Consequently, the false alarm rate increases. This is often the case for steep seamounts, since reverberation quickly drops off following the echo from the seamount. Fig. 1 shows an example of this using modeled data. A seamount rises into the sound channel resulting in a strong increase in the reverberation followed by a noise limited region.

C. Predicting FARI

Predicting FARI requires an acoustic model capable of estimating the expected power of received reverberation. A valid sound-speed profile and detailed terrain model are required as input to the acoustic model. An account of how reverberation is estimated is found in [29].

The probability of a false alarm being generated at a nontarget position equals the probability that the normalized data at the corresponding sample exceeds the selected threshold

\[
P_{fa[i]} = \Pr \{ s[i] > h \} \tag{5}
\]
where \( h \) is the threshold. Inserting (1) yields

\[
P_{fa}[\hat{\lambda}] = \Pr \left\{ \frac{\Lambda[\hat{\lambda}]}{\mu[\Lambda[\hat{\lambda}]]} > h \right\} .
\]  

(6)

The reverberation power is exponentially distributed and therefore can be expressed as

\[
\Lambda[\hat{\lambda}] = \lambda[\hat{\lambda}] E[\hat{\lambda}]
\]

(7)

where \( \lambda[\hat{\lambda}] \) is a deterministic variable and equals the expectation of \( \Lambda[\hat{\lambda}] \). \( E[\hat{\lambda}] \) is an exponentially distributed random variable with a probability density function given by

\[
f_{E[\hat{\lambda}]}(E[\hat{\lambda}]) = \begin{cases} \exp(-E[\hat{\lambda}]), & E[\hat{\lambda}] \geq 0 \\ 0, & E[\hat{\lambda}] < 0 \end{cases}
\]

(8)

Inserting (7) into (6)

\[
P_{fa}[\hat{\lambda}] = \Pr \left\{ \frac{\Lambda[\hat{\lambda}]}{\mu[\Lambda[\hat{\lambda}]]} > h \right\} .
\]  

(9)

Conditioning on \( \mu[\Lambda[\hat{\lambda}]] \) and recognizing that \( E[\hat{\lambda}] \) is an exponentially distributed random variable [6, Ch. 6.3], then (9) becomes

\[
P_{fa}[\hat{\lambda} | \mu[\Lambda[\hat{\lambda}]]] = \exp \left( -h \frac{\lambda[\hat{\lambda}]}{\mu[\Lambda[\hat{\lambda}]]} \right) .
\]  

(10)

Inserting (3) and (7)

\[
P_{fa}[\hat{\lambda} | \mu[\Lambda[\hat{\lambda}]]] = \exp \left( -h \sum_{j=-L/2}^{L/2} \frac{c[j] \lambda[i-j] E[i-j]}{C \lambda[\hat{\lambda}]} \right) .
\]  

(11)

Conditioning on \( \mu[\Lambda[\hat{\lambda}]] \) and recognizing that \( E[\hat{\lambda}] \) is an exponentially distributed random variable [6, Ch. 6.3], then (9) becomes

\[
P_{fa}[\hat{\lambda} | \mu[\Lambda[\hat{\lambda}]]] = \exp \left( -h \sum_{j=-L/2}^{L/2} \frac{c[j] \lambda[i-j] E[i-j]}{C \lambda[\hat{\lambda}]} \right) .
\]  

(11)

Note that \( c[j] \) should be zero to ensure that the analyzed sample is not included in the background estimate. The conditioning on \( \mu[\Lambda[\hat{\lambda}]] \) can be removed by taking the expectation over \( \mu[\Lambda[\hat{\lambda}]] \). Since \( \mu[\Lambda[\hat{\lambda}]] \) is a random variable dependent on the random variables \( E[\hat{\lambda}] \) for \( i \in [-(L/2), (L/2)] \), the expectation is an integral over each \( E[\hat{\lambda}] \)

\[
P_{fa}[\hat{\lambda}] = \prod_{j=-L/2}^{L/2} \left\{ \int_{-\infty}^{\infty} \exp \left( -h \frac{c[j] \lambda[i-j] E[i-j]}{C \lambda[\hat{\lambda}]} \right) \times f_{E[i-j]}(E[i-j]) dE[i-j] \right\} .
\]  

(12)

Inserting (8)

\[
P_{fa}[\hat{\lambda}] = \prod_{j=-L/2}^{L/2} \left\{ \int_{-\infty}^{\infty} \exp \left( -h \frac{c[j] \lambda[i-j] E[i-j]}{C \lambda[\hat{\lambda}]} \right) \right. 

\]  

\[
- E[i-j] \left. \right) dE[i-j] \right\} .
\]  

(13)

which is easily seen to be

\[
P_{fa}[\hat{\lambda}] = \left( \frac{1 + \frac{h c[j] \lambda[i-j]}{C \lambda[\hat{\lambda}]}}{1 + \frac{h}{L}} \right)^{-1} .
\]  

(14)

For the CA-CFAR normalizer, without a guard band, \( c[j] = 1 \) for \( j \in [-(L/2), 1] \cup [1, (L/2)] \), and zero otherwise. If the reverberation in the normalization window is stationary and has the same statistical properties as the reverberation in the analyzed sample, so that \( \lambda[\hat{\lambda}] = \lambda[i-j] \) for \( j \in [-(L/2), (L/2)] \), then

\[
P_{fa}[\hat{\lambda}] = \left( 1 + \frac{h}{L} \right)^{-L} .
\]  

(15)

As \( L \to \infty \), \( P_{fa} \) converges to an exponential relationship with the detector threshold \( \exp(-h) \) indicative of perfect normalization. This is as expected when the estimated background is equal to the actual background of the analyzed sample [6, Ch. 7.3]. Therefore, the CA-CFAR normalizer is the ideal normalizer for stationary reverberation.
Modifying (14)

\[ P_{fa}[i] = \exp \left( -\sum_{j=-L/2}^{L/2} \ln \left( 1 + \frac{h \lambda[j][i-j]}{C \lambda[i]} \right) \right). \]  

(16)

Using the Maclaurin series for \( \ln(1+x) \), and discarding second-order terms since \( C \approx \epsilon[i] \) gives

\[ P_{fa}[i] \approx \exp \left( -\sum_{j=-L/2}^{L/2} \frac{h \lambda[j][i-j]}{C \lambda[i]} \right) \]

\[ = \exp \left( -\frac{h \lambda[i]}{C} \sum_{j=-L/2}^{L/2} \epsilon[j] \lambda[i-j] \right). \]  

(17)

Note that this applies only if \( \lambda[i] \) is of at least the same order as \( h \lambda[i-j] \) which should apply in the case of detection.

Let \( \kappa[m,n] \) be the modeled, beamformed, MF reverberation power for bearing-time cell \([m,n]\). The bearing is the center direction of beam number \( m \), steered an angle \( \theta_m \) relative to north. The reverberation is estimated for \( N \) time cells centered at times \( t_n \), and of widths \( \Delta t \). Time cells can be transformed into range cells, assuming a sound speed of \( C \), as follows:

\[ r_n = \frac{c_0 t_n}{2} \]  

(18)

\[ \Delta r = \frac{c_0 \Delta t}{2}. \]  

(19)

Averaged modeled reverberation power within the normalizer window \( \mu_k \) is given by

\[ \mu_k[m,n] = \frac{1}{C} \sum_{j=-L/2}^{L/2} \tilde{\epsilon}[j] \kappa[m-j,n] \]  

(20)

where

\[ \tilde{\epsilon} = \sum_{j=-L/2}^{L/2} \tilde{\epsilon}[j] \]  

(21)

and \( \tilde{L} \) is the number of modeling cells with total width in time equal to the width of the normalization window in (3). \( \tilde{\epsilon}[j] \) is a downsampled version of \( \epsilon[j] \).

\( \lambda[i] \) in (17) can be approximated by the modeled reverberation power. Note that the resolution of the received MF data generally far exceeds the resolution of the modeled reverberation data. This estimate is valid when the received reverberation power is stationary within the model’s resolution cell. Assuming that the acoustic model and its input data are sufficiently accurate, then

\[ \mu_k[m,n] \approx \frac{1}{C} \sum_{j=-L/2}^{L/2} \epsilon[j] \lambda[i-j] \]  

(22)

and

\[ \kappa[m,n] \approx \lambda[i] \]  

(23)

Inserting into (17) falls within the model resolution cell \([m,n]\). Where sample \( i \) falls within the model resolution cell \([m,n]\),

\[ P_{fa}[m,n] \approx \exp \left( -\frac{h \mu_k[m,n]}{\kappa[m,n]} \right) \]  

(24)

where \( P_{fa}[m,n] \) applies for all samples \( i \) within the model resolution cell given by \([m,n]\).

In Section II-B, target masking and FARI were described. Occurrence of FARI is predicted by solving the following equation:

\[ \frac{\kappa[m,n]}{\mu_k[m,n]} \geq T \]  

(25)

where \( T \) is a selected threshold. Resolution cells that satisfy this inequality are henceforth called FARI areas. Note that the threshold \( T \) is different from the detector threshold in (6). This threshold is used to single out the model resolution cells with a significant probability of false alarm. In principle, all resolution cells could be taken into account, but this would not be practical as the computational effort of the method would increase significantly.

Fig. 1 illustrates the stepwise procedure for finding FARI areas. First, an acoustic raytrace model is run in a single direction. The acoustic model outputs reverberation and noise. Reverberation is normalized and a threshold is applied. Each peak exceeding the threshold signifies a FARI area. A noise floor of 69 dB re 1 \( \mu \)Pa is used. This is the noise received in the beam and contains both ambient noise and self noise.

Assume that there are \( J \) FARI areas. From (24), the probability that a false alarm is generated in a sample in a FARI area in the resolution cell given by \([m^{(j)},n^{(j)}]\) is given by

\[ P_{fa}[m^{(j)},n^{(j)}] \approx \exp \left( -\frac{h \mu_k[m^{(j)},n^{(j)}]}{\kappa[m^{(j)},n^{(j)}]} \right) \]  

(26)

Due to the inherent localization error of sonars, false alarms may be displaced from the FARI area. The localization error can be divided into a bearing error and a range error. The range and bearing errors are assumed Gaussian and zero mean with standard deviations \( \sigma_\theta(\theta) \) and \( \sigma_r(\delta) \), respectively. These standard deviations depend on the sonar system used and the accuracy of the sound-speed measurements used in the localization. The bearing dependence of the bearing error applies for towed array systems, while the range dependency applies for all sonar systems. The bearing error for a single false alarm is given by [31]

\[ \sigma_\theta(\theta) = \frac{\beta(\theta)}{\sqrt{s}} \]  

(27)

where \( \beta \) is the 3-dB beamwidth of the receiver, and \( s \) is the signal-to-reverberation-and-noise ratio of the echo. However, the measured quantity \( s \) is not available when predicting the probability of false alarm, therefore a constant is used instead. Farina and Hanle [31] suggest using \( s = 100 \)

\[ \sigma_\theta(\theta) = \frac{\beta(\theta)}{10}. \]  

(28)

The range error is assumed to be increasing linearly with range due to an error in the sound-speed measurements

\[ \sigma_r(\delta) = av \]  

(29)

where
where \( r \) is the estimated range of the echo, and \( \alpha \) is a constant that depends on the accuracy of the sound-speed measurements.

Assume that these errors far exceed any other positional errors in either modeled or recorded data used. \( f_j(\theta, r) \) is then the probability density function for the location of a false alarm generated by the \( j \)th FARI area located in \((\theta_j, r_j)\).

\[
f_j(\theta, r) = \frac{1}{2\pi\sigma_\theta(\theta)\sigma_r(r)} \exp\left(-\frac{(\theta-\theta_j)^2}{2\sigma_\theta^2(\theta)} - \frac{(r-r_j)^2}{2\sigma_r^2(r)}\right). \tag{30}
\]

Note that \((\theta_j, r_j)\) are discrete bearings and ranges used in the acoustic modeling and the position of FARI area \( j \), while \((\theta, r)\) are continuous spatial variables.

To determine the probability that a false alarm is generated in a small area around \((\theta, r)\) with widths \( \Delta R \) and \( \Delta \theta \), (30) must be integrated over that small area and then multiplied by \( P_{fa}[m(\cdot), n(\cdot)] \). This probability \( F_j \) is given by

\[
F_j(\theta, r) = P_{fa}[m(\cdot), n(\cdot)] \times \int_{\theta-\Delta\theta/2}^{\theta+\Delta\theta/2} \int_{r-\Delta R/2}^{r+\Delta R/2} f_j(\theta', r') \, dr' \, d\theta'. \tag{31}
\]

The total probability for a false alarm appearing in a sample in the small area around \((\theta, r)\), taking into account all FARI areas, is given by

\[
MP_{fa}(\theta, r) = 1 - \prod_{j=0}^{J-1} (1 - F_j(\theta, r)). \tag{32}
\]

If the small area defined by \( \Delta R \) and \( \Delta \Theta \) corresponds to a single sample in the recorded sonar MF data, then \( MP_{fa}(\theta, r) \) is the estimated probability that a false alarm appears in a single sample. This corresponds to the probability of false alarm (PFA) [5, Ch. 12.1]. The metric defined in (32) is henceforth called the modeled probability of false alarm \( MP_{fa} \). For \( MP_{fa} \) to match the true PFA of a sonar system, accurate reverberation modeling and detailed environmental input are required, and other false alarm rate increasing effects, such as clutter, must also be taken into account.

### III. Results

The data used in this section were recorded during the 2002 sea trial in the NAT III project, in an experiment called Clutter Experiment 02 (CEX02).

The sonar system consists of a towed source and linear array. The array has 64 triplet hydrophones spaced at half a wavelength [32]. The source and the receiver were deployed at 74- and 92-m depth, respectively. The source transmitted hyperbolic frequency-modulated pulses (HFM) with frequency bandwidths of 800 Hz and durations of 2 s. The received data were beamformed, MF, and normalized before a threshold of 13 dB was applied to find echoes. Conventional line array beamforming was used in addition to cardioid beamforming on the triplet hydrophones for resolving the left–right ambiguity [32]. A Kaiser window of 1000-m width and a \( \beta \) parameter of eight, [30, Ch. 3.1], centered at the analyzed sample was used to estimate the background in the normalizer.

Fig. 2. False alarms recorded on the sonar during a single transmission in CEX02 are overlaid on a topography map based on 50 m × 50 m resolution gridded data. Note that the area only partly covers the sonar coverage. The coordinates are in zone 32 in the Universal Transverse Mercator coordinate system [33]. Examples of seamounts causing increased false alarm rate are labelled A. Examples of ridges causing increased false alarm rate are labelled B. Examples of seamounts and ridges not causing false alarms are labelled C. The dashed line points from the sonar position towards \( C_1 \).

#### A. Observed FARI

The experiment was located close to the shore to maximize sonar clutter. Particularly many false alarms were observed towards the shore.

Fig. 3 shows histograms of the normalized MF data from 20 transmissions for a broadside beam pointing towards shore and a broadside beam pointing towards open sea. Data received before 8 s after transmission are discarded from both beams since the bottom is approximately the same for starboard and port sides of the first 6-km range. The beam pointing towards shore has a total of 164 threshold crossing samples for the 20 transmissions, resulting approximately in a false alarm rate of \( 10^{-4} \). The beam pointing towards open sea has 27 threshold crossings, resulting approximately in a false alarm rate of \( 10^{-5} \). Note that since the source and the receiver are moving, the data on which these rough false alarm rates are estimated is not stationary, but they still clearly show that the false alarm rate is higher close to the shore than in open sea. This is probably due to strongly varying topography and the presence of seamounts causing increased clutter and FARI. Fig. 2 shows recorded echoes from a single transmission, overlaying a topography map of an area east of the vessel. All echoes are assumed to be false alarms. Notice how many of the false alarms are concentrated around seamounts and along steep ridges. Notice also that some seamounts and ridges do not cause concentrations of false alarms. Why some seamounts cause an increased number of false alarms while others do not is not easily determined simply by studying the topography. For example, lack of false alarms at \( C_1 \) is easily explained since it is partly covered by the seamounts labeled \( A_1 \) and \( A_2 \). On the other hand, the absence of false alarms at \( C_2 \) is not as easily understood. A possible reason is that the seamount at \( C_2 \) is smaller and therefore is not ensonified, but
then the ridge at $B_1$ should not have caused false alarms either. Predicting FARI is too complex to assemble by just simply using topography information. This underlines the importance of accurate acoustic modeling to determine what bathymetric features cause FARI, and what features fail in doing so.

### B. Modeled Probability of False Alarm

The modeled probability of false alarm $MP_{fa}$ given by (32) was determined for 20 transmissions using the method described in Section II-C. The source and the receiver moved 4 km southwards during these transmissions.

The reverberation time series were modeled every 1° over the entire sonar coverage with 0.07-s resolution (or 49-m range resolution). A source–receiver depth of 92 m was used in the acoustic modeling, since the acoustic model does not allow depth separation of the source and the receiver. A sound-speed profile measured immediately before starting the experiment was used in the acoustic modeling; see Fig. 4. The terrain model has a 50 m × 50 m resolution. A subset of the same terrain model is shown in Fig. 2. The in-beam noise level in the modeling was set to 69 dB re 1 μPa. A normalizer equivalent to the 1000-m Kaiser window normalizer used on the received data was applied on the modeled reverberation. The threshold in (25) was set to 3 dB. In noise-limited regions and regions with stationary reverberation in the normalizer window, the modeled probability of false alarm is equal to the designed false alarm rate $P_{fa}$.

Figs. 5 and 6 show false alarms from a single transmission overlaying shaded areas that represent areas where $MP_{fa}$ exceeds $10^{-4}$. Some clusters of false alarms coincide well with shaded areas, but there are also numerous examples of absence of false alarms in shaded areas, or absence of shaded areas where there are concentrations of false alarms. The success of the scheme is better evaluated by estimating the probability of false alarm from the recorded data and then making a comparison with $MP_{fa}$. The probability of false alarm can be estimated by counting the number of false alarms $N_{fa}(A)$ in an area $A$ and dividing by the number of samples $N_S(A)$ in the same area. This metric is henceforth called the estimated probability of false alarm, $EP_{fa}$

$$EP_{fa}(A) = \frac{N_{fa}(A)}{N_S(A)}$$

(33)

Let the span of $MP_{fa}$ be divided into $I$ intervals of 0.001 spacing. Let the area $A_i$ be defined as the largest area that satisfies the following condition:

$$0.001i < MP_{fa}(A_i) < 0.001(i + 1)$$

(34)

where $i = 0, 1, 2, \ldots, I - 1$, then $EP_{fa}$ can be determined in area $A_i$ using (33) and can be compared to $MP_{fa}(A_i)$. $EP_{fa}$ for each area $A_i$ is determined for each transmission individually, and then averaged over 20 transmissions. The averaging is necessary to secure a statistically significant number of false alarms in each area $A_i$, but requires that the data set be stationary. Since the source and the receiver moved 4 km during the 20 transmissions, the topography changes from transmission to transmission, resulting in a nonstationary input to the acoustic model, and obviously a nonstationary geographical distribution of false alarms. For the purpose of this study, the probability distributions of false alarms in each area $A_i$ are assumed stationary, since the acoustic model takes into account the varying topography which is also the source of the nonstationary geographical distribution of false alarms. However, note that the size and the location of area $A_i$ are nonstationary. Fig. 7 shows a bar plot of $EP_{fa}$ for different areas with near-constant $MP_{fa}$. As expected, $EP_{fa}$ increases with $MP_{fa}$. A complete false alarm rate model should result in $MP_{fa} = EP_{fa}$, but the modeled probability of false alarm is underestimated by approximately a factor of four.

Figs. 5–7 show that the method does not model the false alarm rate completely, which is expected since only a single source of increased false alarm rate is taken into account: FARI. A second source, which should be significant for a high-resolution sonar in littoral waters, is clutter [11]–[18]. The area where the test was conducted is littered with wrecks and rocky bathymetric
features not covered by the topographic data used in the modeling. Wrecks also cause clutter and increased false alarm rates [4]. This may explain the appearance of concentrations of false alarms in areas with low $MP_{fa}$, but does not explain the observed high $MP_{fa}$ in areas with no false alarms; see Fig. 5. A possible cause for the latter discrepancy is the limited information on the sound speed and bottom types in the area of the test. A single bottom type was used at all ranges in the modeling, but clearly the scattering strengths of the rocky upsslopes close to shore should be higher than the slowly varying bottom close to the sonar vessel; see Fig. 2. The inclusion of range-dependent scatter strength could possibly improve the predictions, but has not been attempted since no detailed map of bottom types is available for this area. Sound-speed profiles were measured from the sonar vessel immediately before and after the experiment, but these measurements do not fully describe the oceanographic variations in the area covered by the sonar. The experiment was conducted close to the outlet of a fjord, which should cause significant horizontal variations in the sound speed. The sound speed was not measured close to the shore during the experiment. Inaccuracies in the sound-speed profile and bottom types used in the acoustic model, for instance due to lack of range-dependence, result in inaccuracies in $MP_{fa}$. Such inaccuracies may explain some of the areas with high $MP_{fa}$ and no recorded false alarms as observed in Figs. 5 and 6.

C. Bottom Depths

The source and the receiver are located in a sound-speed duct that traps a significant amount of acoustic energy (Fig. 4). Some might argue it is therefore sufficient to locate areas with terrain rising into the sound-speed duct, and that these areas generate many false alarms. However, Fig. 8 shows a histogram of bottom depths at false alarm positions from 20 transmissions, approximately 126,000 false alarms in all, 81% of the false alarms are generated at locations with bottom depths below the surface duct, below 120 m. The false alarms are distributed among bottom depth bins of 10-m width. Fig. 9 shows a histogram of false alarms, 35,000 in all, located in areas with $MP_{fa} > 10^{-4}$, where 60% of the false alarms are located in areas with bottom depths below the surface duct. The same bottom depth bin size is used as in Fig. 8. Fig. 10 shows the percentage of false alarms in areas with $MP_{fa} > 10^{-4}$ for different bottom depths. This is essentially the ratio between bin heights in Figs. 8 and 9, and illustrates the success rate of the method for each bottom depth interval. This indicates that the method has a higher success rate within the surface duct than below it, but that it is still able to predict high false alarm rates in areas with bottom depth below the surface duct. Notice also that the model does not successfully predict high $MP_{fa}$ at locations with large bottom depths (> 300 m). Most of the false alarms originating from areas with
more than 300-m bottom depth are located in circles around the sonar at short range and are probably due to short-range bottom bouncing acoustic paths.

D. Terrain Model Resolution

The fidelity of the modeled PFA is clearly dependent on the resolution of the terrain model used. To evaluate the efficacy of the model when using less detailed topographic information, the $MP_{fa}$ is modeled using 100-, 250-, 500-, and 1000-m resolution in the terrain model. All other model and environmental parameters are equal to the ones used in Section III-B.

Fig. 11 is equivalent to Fig. 7 for terrain model resolutions of 50-, 100-, 250-, 500-, and 1000-m. The three most detailed terrain models result in an $MP_{fa}$ that increases steadily with increasing $EP_{fa}$. The two lowest resolutions result in an erratically varying $EP_{fa}$ after the first five intervals. The erratic behavior is partly because the corresponding areas [see (34)] are too small and contain too few false alarms to be statistically significant. The percentage of false alarms located in areas with $MP_{fa} > 10^{-4}$ is 32% for 50-m, 31% for 100-m, 28% for 250-m, 17% for 500-m, and 11% for 1000-m resolution. This indicates that the success of the method is significantly lower for 500- and 1000-m resolutions as compared to the higher resolutions.

Obviously, if no terrain model is available the method is not applicable. This limits the operational usefulness. In its own waters, the Navy often has access to sufficiently detailed terrain models. In enemy waters, this is less likely. However, rapid environmental assessment (REA) missions performed by a submarine or an autonomous underwater vehicle can be launched...
to secure such a data set. For the presented case, a minimum resolution of 250 m is recommended.

IV. CONCLUSION

False alarm rates several orders of magnitude higher than the designed false alarm rate are observed when using active, low-frequency, towed array sonar systems in littoral waters [2], [3]. Possible reasons include clutter and FARI. The latter effect arises when the reverberation in the normalizer window is nonstationary, resulting in an underestimated background level in the normalizer. The increased false alarm rate is observed to be spatially dependent, and appears to depend on local topography. Clusters of false alarms are frequently generated in the proximity of seamounts and underwater ridges [1]–[3].

Acoustic propagation models can be used to map what areas are prone to a raised false alarm rate for a given sonar location and a given environment. Seamounts generate many false alarms, but only if they are ensonified. A method predicting the occurrence of FARI is described and presented. The method combines an acoustic model with high-resolution topographic data to predict the probability of false alarm being generated at a specific range and bearing.

The method is applied on a real data set from an experiment in the NAT III project. The modeled probability of false alarm is shown to compare well with spatial concentrations of recorded false alarms, but the modeling underestimates the probability of false alarm by approximately a factor of four. A possible explanation for the underestimation is that the model does not take into account the raised false alarm rates due to clutter, which may be significant for high-resolution sonars [11], [17], particularly in the presence of wrecks and seamounts [1]. The area is a busy fairway littered with wrecks. A second possible explanation is that the environment used in the acoustic model, specifically the sound speed and bottom type, is inaccurate in some geographic areas. Range dependence in these environmental parameters could increase the performance of the model, however the required data for such an extension is not available for this area. Nonetheless, the presented results show that FARI frequently occurs in littoral areas, particularly in the proximity of seamounts, and that this effect causes a significant increase in the false alarm rate. Most importantly, given a detailed terrain model, this effect can be predicted to some extent.

The operational value of the method depends strongly on available environmental input and the fidelity of the acoustic model. In modern warfare, REA missions may give the required minimum in environmental awareness, but they are still too limited for modeling occurrence of FARI perfectly. Despite these shortcomings the method could be implemented in an antisubmarine warfare sonar system and thereby reduce the impact of FARI. An example of such an implementation is to use the method to determine a normalizer window size that minimizes the modeled probability of false alarm, for example, by excluding samples in the normalizer window that, according to the method, contribute to an underestimated background power. However, one should be careful that the implementation does not impede the detection of a present target, for instance, due to the observed discrepancies between modeled probability of false alarms and concentrations of measured false alarms observed; see Fig. 5. This can be assessed by testing the method on data sets containing one or more targets and then comparing the false alarm rate and the target signal-to-noise-ratio with the results from traditional normalization schemes. The proposed implementation is an example of how superior environmental knowledge may augment the use of naval sonars in antisubmarine warfare, and is suggested as future work.

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Paper 3

Target depth estimation using a ray backpropagation scheme on mid-frequency, active sonar data
Target depth estimation using a ray backpropagation scheme on sonar data – simulations and experiments

Karl Thomas Hjelmervik *

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Abstract

Classification is one of the main challenges in anti–submarine warfare using active sonars in littoral waters. Sea mounts and rocky ridges may result in large numbers of false alarms, some exhibiting submarine–like behaviour. High false alarm rates result in complex tactical pictures. Efficient classification tools for reducing the amounts of false alarms are needed.

The presented method determines a target depth probability density function by applying ray backpropagation on data from a mid–frequency, active sonar. This information may be used for classification, for instance by prioritising targets with probable depths above the sea floor.

The fidelity of the method is studied by simulating scenarios with different environments at different target ranges and depths. For sufficiently high target echo signal–to–noise ratios, the method is shown applicable for ranges up to 15 km.

Finally the method is tested on recorded data for targets located on the sea floor and in the upper half of the water column. The method successfully estimates the target depth with an accuracy sufficient for classification purposes 82% of the time. Averaged over all measurements the target depths are estimated within 40 m of the true depth.

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1 Introduction

One of the main challenges in anti-submarine warfare using active sonars in littoral waters, is classification. Varying topography and rocky outcrops result in a large number of false alarms [1–4], some exhibiting submarine-like behaviour [2]. High number of false alarms results in a complex tactical picture presented to the sonar operator. Efficient classification tools for reducing the amounts of false alarms are needed. This work presents a method that estimates target depth. Knowledge of target depth is a powerful classification clue which allows separation of targets located on the sea floor from other targets.

Matched-field processing (MFP) is a widely used technique for estimating target location, and is mostly used for passive sonars [5,6]. MFP has also successfully been extended to low-frequency, active sonars [7] showing promising results for target depth estimation. An alternative method for target location is ray backpropagation [8–10]. This method has been demonstrated on recorded data from low-frequency passive sonar systems [8,10]. Ray back-propagation is a method where acoustic rays are traced in a direction defined by the angle of arrival measured on the sonar. In the passive sonar case, multiple arrivals are needed. Locations where rays representing the different arrivals intersect are candidate target locations.

In the present work, ray backpropagation is used on data from a mid-frequency, active sonar in order to determine the depth of a detected target. The main advantage of active systems compared to passive systems, is that the arrival time is known. The arrival time is the one-way propagation time between the sonar and target. A requirement on the sonar system is that it has sufficient aperture to beamform the received signals with relatively narrow vertical beamwidths. The exact requirements on the beamwidths depend on the target range, signal-to-reverberation and noise levels (SNR), and the required accuracy of the results. Beamforming allows estimation of optimal arrival angles, that is the horizontal and vertical direction in which the target signal level is strongest. By tracing a ray in this direction for a time equal to the measured travel time, the target depth may be estimated. The accuracy of the estimate depends on what detail the environment is known, the sonar aperture and depth, the depth and range of the target, and the received signal strength from the target.
2 Method

The target is detected using standard sonar processing; beamformer, matched filter, and detector. An omnidirectional signal is transmitted, but the received signal is beamformed for a discrete set of horizontal and vertical steering angles. Let the received signal from the target after beamforming, matched filtering, and normalisation be given by $x(\theta, \phi, t)$, where $\theta$ is the horizontal angle, $\phi$ is the vertical angle, and $t$ is the arrival time. The measured optimal arrival angles, $\hat{\theta}_m$ and $\hat{\phi}_m$, and arrival time, $\hat{t}_m$ are determined by maximising $x(\theta, \phi, t)$. Assuming cylindrical symmetry, the vertical angle is the only angle containing information on target depth and range. Due to noise, the maximum achievable vertical angle resolution is given by the standard deviation of the vertical optimal arrival angle, $\sigma_\phi$ [12]:

$$\sigma_\phi = \frac{\phi_{BW}}{\sqrt{s}} \quad (1)$$

$\phi_{BW}$ is the vertical beam width. $s$ is the linear SNR. The measured vertical angle is therefore considered a random process. Assuming a Gaussian distribution with standard deviation given by (1), the probability density functions for $\hat{\phi}_m$ is given by:

$$f_{\phi}(\hat{\phi}_m) = \frac{1}{\sqrt{2\pi}\sigma_\phi} \exp\left(-\frac{(\hat{\phi}_m - \phi_m)^2}{2\sigma_\phi^2}\right) \quad (2)$$

Note that measured time of arrival also has an uncertainty. For broadband signals this uncertainty has low impact on the target location estimate compared to errors in arrival angle. Measured arrival time is therefore assumed approximately equal to the true arrival time, $t_m$:

$$\hat{t}_m \approx t_m \quad (3)$$

If narrowband signals are used, uncertainty in arrival time should be included.

For a known environment, let a raytracer [11] be defined by the vector function $g$ as follows:

$$\begin{bmatrix} r \\ z \end{bmatrix} = g(\phi, t, m) \quad (4)$$
r and z are the range and depth of the end point of a ray with initial vertical angle φ and travel time t. m is a vector containing information on the environment. Target range, \( r_t \), and depth, \( z_t \), are estimated by propagating a ray in a direction defined by the optimal arrival angles and a travel time equal to the arrival time, \( t_m \). The target range and depth is then given by:

\[
\begin{bmatrix}
\hat{r}_t \\
\hat{z}_t
\end{bmatrix} = g(\hat{\phi}_m, t_m, m)
\] (5)

Since \( \phi \) is a random process, target range and depth are also random processes. The vertical angle is not linearly related to the resultant range and depth, the range and depth may therefore not be assumed Gaussian distributed. Their distributions may be determined numerically by tracing rays for all vertical angles with significant probability densities. In this study all angles within two standard deviations of the measured optimal arrival angle, are traced. More than two standard deviations have been tested without any significant changes to the results. Let \( N \) rays have initial angles distributed homogeneously within the interval \( [\hat{\phi}_m - 2\sigma_\phi, \hat{\phi}_m + 2\sigma_\phi] \). The separation in angle between neighbouring rays is then:

\[
\Delta \phi = \frac{4\sigma_\phi}{N - 1}
\] (6)

and their angles are given by:

\[
\phi[j] = \hat{\phi}_m + j\Delta \phi
\] (7)

where \( j \in [-\frac{N-1}{2}, \frac{N-1}{2}] \). Tracing these rays results in range–depth pairs given by:

\[
\begin{bmatrix}
\hat{r}[j] \\
\hat{z}[j]
\end{bmatrix} = g(\phi[j], t_m)
\] (8)

Each ray represents an angle interval of \( \Delta \phi \) width. The probability that the true optimal arrival angle is within such an interval is given by:

\[
P_j = \int_{\phi[j] - \frac{\Delta \phi}{2}}^{\phi[j] + \frac{\Delta \phi}{2}} f_\phi(\phi) \, d\Phi
\] (9)

Assuming that \( f_\phi(\phi) \) is uniform within \( [\phi[j] - \frac{\Delta \phi}{2}, \phi[j] + \frac{\Delta \phi}{2}] \), then:

\[
P_j \approx f_\phi(\phi)\Delta \phi
\] (10)
The water column is divided into K intervals of equal size, $\Delta z$, and centred at depth $z_k$. The probability that the target is within one such interval equals:

$$Q_k(z_k) = \sum_{j=0}^{N-1} P_j \left( H\left(z[j] - \left(z_k - \frac{\Delta z}{2}\right)\right) - H\left(z[j] - \left(z_k + \frac{\Delta z}{2}\right)\right) \right)$$  \hspace{1cm} (11)

where $H$ is the Heaviside–function, which equals unity for non–negative arguments and zero otherwise. The probability density function for the target depth is then approximated as:

$$f_z(z_k) \approx \frac{Q_k(z_k)}{\Delta z}$$  \hspace{1cm} (13)

Target depth mean– and MAP–estimates [13] are defined as:

$$z_{\text{mean}} = \sum_{k=0}^{K-1} z_k f_z(z_k) \Delta z$$  \hspace{1cm} (14)

$$z_{\text{MAP}} = \max_{z_k} \left( f_z(z_k) \right)$$  \hspace{1cm} (15)

and the associated variance is given by:

$$\sigma_z^2 = \sum_{k=0}^{K-1} (z_k - z_{\text{mean}})^2 f_z(z_k) \Delta z$$  \hspace{1cm} (16)

Mean estimates, for targets located close to the sea floor, are generally pulled away from the sea floor due to reflectivity of the sea floor. This makes the mean estimate a poor choice for separating real targets from false alarms generated at the sea floor. The MAP–estimate has no obvious weaknesses in that regard, and is therefore used in the present work.

2.1 Multiple–ping information

The resultant target depth probability density function in (13) is estimated from single–ping information. Three different techniques on how the estimate can be extended to include information from multi–ping measurements are proposed: smoothing of the vertical optimal arrival angle, averaging the target depth probability density function, and depth tracking.
Smoothing of the vertical optimal angle is done in the following way:

\[ \phi_s = \frac{\sum_{n=0}^{N-1} \hat{\phi}_m[n] \sigma^{-1}_\phi[n]}{\sum_{n=0}^{N-1} \sigma^{-1}_\phi[n]} \]  

where \( \phi_s \) is the smoothed vertical optimal arrival angle. The arrival time is smoothed in the same manner:

\[ t_s = \frac{\sum_{n=0}^{N-1} \hat{t}_m[n] \sigma^{-1}_t[n]}{\sum_{n=0}^{N-1} \sigma^{-1}_t[n]} \]  

The smoothed vertical optimal arrival angle and arrival time are used in estimating the target depth probability density function. The main advantage of this technique is the reduction of the standard deviation of the vertical optimal arrival angle. Assuming equal standard deviation in all measurements, then the standard deviation, \( \phi_s \), of the smoothed angle is given by:

\[ \sigma_{\phi_s} = \frac{\sigma_{\phi}}{\sqrt{N}} \]  

Smoothing is only valid when the statistical properties of the angle are stationary. In most cases, the sonar and target move relative to each other, causing the sonar–target geometry to change from ping to ping. Non–stationarity is also caused by a changing environment, such as a time–or location–dependent sound speed profile. The vertical optimal arrival angle is therefore generally non–stationary. However, if the target depth remains constant and range variations within the smoothing window are small compared to target range, then the geometry may be assumed stationary. The environment may be monitored by frequent sound speed measurements, but in most cases changes to the sound speed profile in order of minutes are negligible. A few minutes of recordings are sufficient for smoothing.

In cases where the sonar–target geometry or the environment is not stationary, averaging the target depth probability density function can be considered instead of smoothing the vertical optimal arrival angle. The target depth is still required to be unchanging, but the other geometric parameters, as well as the environment, may be non–stationary. The averaged version of the target depth probability density function, \( h_{z_k} \), is given
by:

\[ h_z(z_k) = \frac{\sum_{n=0}^{N-1} f_z^{(n)}(z_k) \sigma_z^{-1}[n]}{\sum_{n=0}^{N-1} \sigma_z^{-1}[n]} \] (20)

\( f_z^{(n)} \) is the target depth probability density function for ping number \( n \). \( h_z \) is the averaged target depth probability density function.

Smoothing the vertical optimal arrival angle may also be combined with averaging the depth probability density function. For instance, if, during a ten–ping period, the geometry and environment is considered sufficiently stationary to smooth the vertical optimal angle, a ten–ping smoothing of the vertical optimal arrival angle may be applied before estimating the target depth probability density function. The resultant density functions for each ten–ping interval are then averaged to find a final target depth probability density function based on all ping–information.

Three–dimensional tracking is another way of exploiting multiple–ping information. Conventional tracking algorithms for active sonars take into account two dimensions only. Including target depth in tracking algorithms allows tracking of submarines manoeuvring through areas with many false alarms generated by the sea floor. For example, a track on a submarine may be separated from a track on an oil pipeline the submarine runs along. Such an algorithm requires accurate target depth estimates, and should therefore only be considered for sonar systems with low vertical beamwidths. In the current work the measured data is considered to be of too low resolution for using a full tracking approach. Instead, a simplified depth tracker algorithm is used. The algorithm tracks the target depth MAP–estimate and removes measurements where the change in the MAP–estimate from the previous MAP–estimate exceeds a selected threshold. The selected threshold should depend on total time between measurements and on the expected depth change rate of the target. Note that removed measurements should result in initiation of a new track at a new depth. If the sudden change in MAP–depth is due to measurement error, the track quickly expires due to lack of new measurements with comparable target depth MAP–estimates.
Figure 1: Measured sound speed profile in the Norwegian trench at the time of the sea trial.

3 Experimental data

A sea trial was conducted in the Norwegian trench at approximately 60° North and 4° East. The area is virtually flat with a sea floor depth of 300 m. The sound speed profile was measured during the trial, see Fig. 1.

The vessel was equipped with a hull-mounted sonar, at 7 m depth, working on frequencies below 10 kHz. The sonar transmitted hyperbolic FM pulses with 2 kHz bandwidth and 1 s pulse length. The vertical beamwidth is 15°.

Several oil pipelines crisscross the area of the sea trial. An echo repeater was also in the area. The sonar detected and maintained tracks on both the echo repeater and the pipelines. The measured arrival time and vertical optimal arrival angle for two tracks on the pipelines (PL1 and PL2) and
Figure 2: Measured arrival time and vertical optimal arrival angle as a function of five–ping interval number for PL1 (solid), PL2 (dashed), and ER (dotted). The measurements are smoothed in five–ping wide rectangular windows.

one on the echo repeater (ER) are shown in Fig. 2. The depths of the three targets are listed in Table 2. The measured arrival times and vertical optimal arrival angles were smoothed using a five–ping wide window as described in section 2.1.

4 Simulated scenarios

The proposed methods validity is tested in various environments and sonar–target geometries. The acoustic raytrace model Planeray [14] is used to simulate arrival times and angles of echoes from targets at different ranges and depths. Arrival angles between -20° and 20° are considered.
Sixteen test cases are studied in all, see table 1. The measured sound speed profile is shown in Fig. 1. The sea floor is at a constant 300 m depth in all test cases. The sonar used in the modelling is described in 3. Thirty detections per range step are simulated in each test case. Zero–mean Gaussian noise with standard deviation given by (1) is added to the simulated vertical optimal arrival angle:

\[ \hat{\phi} = \phi + g(\sigma_\phi) \]  

For SNRs of 13.5 dB and 23.5 dB the standard deviations of the measured vertical angles are 3.3° and 1°, respectively.

Target depth MAP–estimates and standard deviations are determined for each test case using the method described in section 2. The results are presented in Figs. 3 and 4, and table 1. Smoothing and decimation of arrival time and vertical optimal arrival angles are applied using a ten–ping rectangular window. Estimates with no smoothing applied are also included for reference, but not included in the plots. The MAP–estimate is taken from the target depth probability density function averaged over all pings; 30 pings when no smoothing is applied, and three when smoothing and decimation is applied.

Observe from Figs. 3 and 4 that, for increasing ranges, the standard deviation seems to converge towards 87 m, which is the standard deviation estimated from a uniform target depth distribution. The standard deviation is a good quality indicator of the target depth estimate. Estimates with standard deviation above a selected threshold should be disregarded. The selected threshold should depend on the application. For the cases presented here 84% of the estimated target depths falls within 50 m of the actual target depth when the standard deviation is lower than 70 m. This accuracy is sufficient for classification purposes where the main goal is separating targets located on the sea floor from other targets. Table 1 shows the maximum range and number of range steps with standard deviations lower than 70 m for each test case.

In all test cases, high–SNR detections (23.5 dB) result in improved target depth estimates and lower target depth standard deviations as compared to low–SNR (13.5 dB) detections. Smoothing the measured data is better use of multiple–ping information than averaging target depth probability density functions.

Increasing ranges mostly result in decreased accuracy in target depth estimates and higher standard deviations, see Fig. 3 and 4. A notable
exception is the high target depth standard deviations and poor target depth estimates at ranges between 3 km and 6.5 km for a target at 300 m depth when using the measured sound speed profile, see Fig. 4. The sound speed maximum at approximately 220 m depth is the cause, see Fig. 1. Fig. 5 shows how a group of rays are split in two due to the sound speed maximum. Some rays hit the sea floor, while others refract towards the surface. Consider a target on the sea floor, and let the thick ray represent the true propagation path to the target. Small errors in the vertical optimal arrival angle may result in estimated target depths close to the surface. This results in ambiguities as shown in Fig. 6. This ambiguity strongly influences the estimated standard deviation as seen in the peak in the standard deviation for ranges around 5 km for the target located on the sea floor, see Fig. 4. Smoothing the vertical optimal arrival angle reduces the ambiguity, see Fig. 6.

5 Results and discussion

The described method is tested on data recorded during the sea trial described in section 3. Received hydrophone data are processed with standard beamforming, matched filtering, and normalisation. The horizontal and vertical optimal arrival angles are determined for each ping by using conventional beamforming for different horizontal and vertical steering angles. The arrival time and the vertical optimal arrival angles are smoothed in five–ping windows as described in section 2.1, see (17).

Figs. 7 and 8 show the estimated target depth probability density functions for PL1 and PL2. The probability densities are averaged as described in section 2.1, see (20). Table 2 shows target depth MAP–estimates and standard deviations for each target. The depths of PL1 and PL2 are both estimated to be close to the sea floor. The target depth standard deviation is significantly lower for PL1 than for PL2. This is expected since the range to PL1 is never greater than 2.6 km, while PL2 has a maximum range of 4.2 km. The ranges are estimated from the arrival times presented in Fig. 2 assuming a constant sound speed of \(1474 \text{ m s}^{-1}\). According to the simulations presented in section 4, the method should give accurate target depth estimates with low standard deviations for a target located at the sea floor and at PL1s ranges, see Fig. 4. PL2s range extends into the region where a local peak in the standard deviation was observed, as discussed in section.
Figure 3: Estimated target depth (left) and standard deviation (right) as a function of target range for high (red) and low (blue) SNR for the test cases using the constant sound speed profile. True target depth is indicated by the dashed line in the left plots. Ten-ping smoothing of the vertical optimal arrival angle is applied.

4. This effect is the main contributor to the higher standard deviation in the estimated target depth for PL2 than for PL1, and is also the cause for the extra peak at 50 m depth in the target depth probability density function for PL2.

Fig. 9 shows the estimated probability density function for the depth of ER averaged over all pings. The target depth MAP-estimate and standard deviation are listed in table 2. The estimate is close to the true depth of 81 m. The target range is between 4 and 5.5 km for all pings. Observe from the simulations presented in Fig. 4 that target depth is typically underestimated for these ranges, as is also the case for the recorded data.
Despite the difficulties introduced by the complex measured sound speed profile, the depth of the three targets are all estimated with an accuracy sufficient for target classification. Both targets located on the sea floor are easily seen to be either objects on the sea floor or a target moving very close to the sea floor. The depth of the echo repeater is successfully estimated close to the sea surface. This is a strong classification clue. An echo with very low probability of originating from the sea floor has a high probability of being a submarine.

In anti–submarine warfare operations, classification is required after a few pings. Fig. 10 shows the estimated target depth probability density
Figure 5: A set of rays traced for 4.2 km at initial vertical angles between 4° and 8° in 0.5° steps. The thick ray has a 6° initial vertical angle and traces the path to a target located on the sea floor at 4.2 km range.

function as a function of ping number for ER. The five–ping smoothing of the recorded data is still used, resulting in a five–ping lag, but this is considered tolerable for anti–submarine warfare operations. Consider a division of the water column into three regions. For the given example define the shallow region from 0 to 100 m depth, mid–depth region from 100 to 200 m depth, and deep region from 200 to 300 m depth. Knowing which region a detected target is located in is a strong classification clue. For the three targets considered, the method locates the target in the correct region in 82 % of the five–ping intervals.

The target depth estimates are based on multi–ping recordings, which renders the update rate too low for allowing the results to be included in conventional sonar tracking algorithms. Similar estimates may be given
Figure 6: Estimated probability density distribution for the depth of a target located on the bottom with a ten–ping smoothing of the vertical optimal arrival angle (dashed line) and without smoothing (solid line). For single–ping recordings, but estimated uncertainties are too high for depth estimates to be useful in the tracking. However, a simplified tracker, which tracks the target depth only, is useful. Such a tracker could disqualify measurements that results in sudden leaps in target depth. One such leap is observed in Fig. 11. Notice that after the seventh five–ping interval the MAP–estimate for the target depth leaps from 250 m depth to 50 m depth, before dropping down to 250 m again at the tenth five–ping interval. Fig. 8 shows how a simple depth tracker resolves the ambiguity and improves the estimated target depth probability density function. The depth tracker is described in section 2.1. A threshold of 100 m is used. The MAP–estimate remains the same, but the standard deviation is reduced from 60 m to 40 m.
6 Conclusion

A method capable of estimating the depth of a submerged target using mid-frequency, active sonar data is demonstrated. The methods input is the present environment and recorded arrival times and vertical optimal arrival angles.

Simulated data show that the method is able to estimate target depths at ranges up to 15 km. The maximum range depends on the environment and the sonar parameters, particularly the vertical beamwidth of the receiver. Generally, the uncertainty of the estimate increases with range and decreases for increasing SNR. Sound speed maxima are shown to cause ambiguities in the estimated target depth for certain target ranges, result-
Figure 8: The estimated target depth probability density function for PL2 averaged over all pings using five–ping smoothing of the vertical optimal arrival angle (solid line). The target depth probability density function after applying a simplified tracking algorithm is also shown (dashed line). True target depth is 300 m.

ing in local maxima in target depth standard deviation for these ranges. Smoothing the vertical optimal arrival angle, reduces the uncertainty in angle and results in less ambiguity. Simulations help predict what target depths and ranges are difficult to estimate. Estimated standard deviation is a good indicator of the quality of estimations. Decent MAP–estimates were observed as long as the estimated standard deviation was lower than 70 m, which is 80% of the standard deviation estimated for a uniform target depth distribution.

The method is applied on a data set containing acoustic returns from two pipelines located on the sea floor at 300 m depth, and an echo repeater
located at 81 m depth. Averaged over all recorded pings, the depth of the pipelines are estimated to 290 m and 260 m depth, and the echo repeater to 50 m depth. The target depth estimate was acquired for every five pings, whereof 82% of the five-ping intervals resulted in estimates considered sufficiently accurate for classification purposes. A simplified depth tracking algorithm is shown to resolve ambiguities observed in the estimated target depth probability density functions. The depth tracking algorithm also lowered the standard deviation, but the MAP-estimate remained unchanged.

Knowing the depth of a target is a powerful tool for classification of submarines and mine-like objects in the ocean. Coupled with other classification information, such as target heading and speed or target Doppler
Figure 10: The estimated target depth probability density function for ER for each five–ping interval. True target depth is 81 m.

speed, the false alarm rate in littoral areas may be reduced significantly.

References


Figure 11: The estimated target depth probability density function for PL2 for each five–ping interval. True target depth is 300 m.


Table 1: A list of test cases and what sound speed profiles (SSP), target depths, SNRs were used, and whether smoothing was used. All test cases used target ranges between 1 km and 15 km in 500 m steps. The maximum achieved range (max range) and percentage of range steps (OK ranges) with estimated depth standard deviation below 70 m are also listed.

<table>
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<th>Depth [m]</th>
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<th>Smoothing</th>
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<th>OK ranges [%]</th>
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Table 2: MAP estimates and standard deviations (STD) of target depths for the targets considered.

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<th>STD [m]</th>
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<td>PL2</td>
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<tr>
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Paper 4

Target depth estimation using a ray backpropagation scheme on mid-frequency, active sonar data
Target depth estimation using a ray backpropagation scheme on mid-frequency, active sonar data

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Classification is one of the main challenges in anti–submarine warfare using active sonars in littoral waters. Sea mounts and rocky ridges may result in large numbers of false alarms, some exhibiting submarine–like behaviour. High false alarm rates result in complex tactical pictures. Efficient classification tools for reducing the amounts of false alarms are needed.

Conventional active sonar processing outputs a target’s range and bearing. The presented method gives an estimate of the target depth on basis of data from a mid–frequency, active sonar. The vertical arrival angle is determined by beamforming received sonar data vertically. Ray backpropagation is combined with Bayesian inversion to estimate the a posteriori target depth probability density function. Empirical orthogonal functions (EOF) are used to represent the sound speed profile. EOF coefficients and sonar depth are included in the inversion in order to improve the target depth estimate.

The method is tested on recorded data from two targets; one located at 300 m depth and one at an unknown depth in the upper 81 m of the water–column. Target depth is estimated to 285 m depth and 45 m, respectively, which is sufficient for classification purposes in anti–submarine warfare.

1 Introduction

One of the main challenges in anti–submarine warfare using active sonars in littoral waters is classification. Varying topography and rocky outcrops result in a large number of false alarms [1–4], some of which exhibit very submarine–like behaviour [2]. High number of false alarms results in a complex tactical picture. Efficient classification tools for reducing the amounts of false alarms are therefore needed. This work presents a method that estimates target depth. Knowledge of target depth is a powerful classification clue which allows separation of false targets located on the sea floor, such as wrecks, sea mounts, and large boulders, from true targets.

Matched–field processing (MFP) is a widely used technique for estimating target location, and is mostly used for passive sonars [5, 6]. MFP has also successfully been extended to low–frequency, active sonars [7] showing promising results for target depth estimation. Bayesian inversion is an extension of MFP that also outputs the probability density function for the model parameters [8]. Collins and Kuperman [9] introduced focalisation as a means for improving target depth estimates by including environmental parameters, e. g. sound speed profiles, in the inversion process. An alternative method for target localisation is ray backpropagation [9–11]. This method has been demonstrated for low–frequency, passive sonar systems [9, 11]. Ray backpropagation is a method where acoustic rays are traced in a direction defined by the angle of arrival measured on the sonar. In the passive sonar case, multiple arrivals are needed. Locations where rays representing the different arrivals intersect are candidate target locations.

This paper presents a method that estimates the a posteriori target depth probability density function of a detected target. Vertical arrival angles and arrival times are measured using a mid–frequency, active sonar. Bayesian inversion and ray backpropagation are used to estimate probable target depths. Focalisation is employed by including sonar depth and the sound speed profile in the inversion process. Empirical orthogonal function (EOF) coefficients represent the sound speed profiles [9] in the inversion.

The method is applied on a data set containing detections of two different targets. The first target is a pipeline located on the sea floor at 300 m depth. The second target is an echo repeater located at an unknown depth less than 81 m.

2 Method

Let an unknown target be detected using standard active sonar processing; beamforming, matched filtering, and detection [12]. The measured vertical arrival angle is defined as the vertical steering angle maximising the signal–to–noise ratio (SNR) of the target echo. The vertical arrival angle, \( \phi \), and arrival time, \( t \), are considered as Gaussian distributed random processes with standard deviations [13] given by:

\[
\sigma_\phi = \frac{\phi BW}{\sqrt{s}} \\
\sigma_t = \frac{1}{B \sqrt{s}}
\]

(1)

(2)

s is the signal–to–noise ratio of the target echo. \( \phi BW \) is the vertical beamwidth of the sonar. B is the sonar bandwidth.

Let the model parameter vector, \( \mathbf{m} \), contain relevant information on the environment as well as arrival time and vertical arrival angle. Target depth may then be modelled from
\[
d(m) = g(m) \tag{3}
\]
g is an operator describing the raytracer. \(d(m)\) is the modelled target depth.

Let the elements in \(m\) be samples from independent random processes, and let the joint probability density distribution for the model parameters be given by \(P(m)\), then:

\[
P_m(m) = P_{m1}(m_1)P_{m2}(m_2) \ldots P_{mL}(m_L) \tag{4}
\]

\(P(m_j)\) is the probability density distribution for the \(j\)th parameter, \(m_j\).

Bayes law states that:

\[
P_{zd}(z|d(m))P_d(d(m)) = P_{dz}(d(m)|z)P_z(z) \tag{5}
\]

\(z\) is the true target depth, and \(d(m)\) is the modelled target depth given by (3). The a posteriori probability density distribution (PPD) for target depth, \(P_{zd}(z|d(m))\), is given by:

\[
P_{zd}(z|d(m)) = \frac{P_{dz}(d(m)|z)P_z(z)}{P_d(d(m))} \tag{6}
\]

Assuming \(P_z(z)\) is uniform, then \(P_d(d(m))\) may be estimated by:

\[
P_d(d(m)) = P_z(z) \int_{-\infty}^{\infty} P_{dz}(d(m)|z)dz \tag{7}
\]

Combining (6-7) gives:

\[
P_{zd}(z|d(m)) = \frac{P_{dz}(d(m)|z)}{\int_{-\infty}^{\infty} P_{dz}(d(m)|z)dz} \tag{8}
\]

\(P_{dz}(d(m)|z)\) is estimated by modelling the target depth for all combinations of model parameters, and assigning a probability to modelled target depth equal to the combined probability of all model parameters, \(P_m(m)\). If multiple model parameter combinations result in the same depth, then the corresponding probabilities are summed. Let the water column be divided into \(K\) intervals, each of \(\Delta z\) width and centred at \(z_k\). \(P_{dz}(d(m)|z_k)\) may then be approximated as a step function with \(K\) intervals where the step heights are given by:

\[
P_{dz}(d(m)|z_k) \approx \frac{1}{\Delta z} \int_m H\left(\frac{\Delta z}{2} - |z_k - d(m)|\right) P_m(m)dm \tag{9}
\]

\(H\) is the unit-step-function, which equals unity for non-negative arguments and zero otherwise. An expression for the PPD can then be found by combining (8–9). Fig. 1 illustrates how the PPD is derived.

Target depth mean– and MAP–estimates [15] are given by:

\[
z_{mean} = \sum_{k=0}^{K-1} z_k P_{zd}(z_k|d(m))\Delta z \tag{10}
\]

\[
z_{MAP} = \max_{z_k} P_{zd}(z_k|d(m)) \tag{11}
\]

and the associated variance is given by:

\[
\sigma^2 = \sum_{k=0}^{K-1} (z_k - z_{mean})^2 P_{zd}(z_k|d(m))\Delta z \tag{12}
\]

Mean–estimates, for targets located close to the sea floor, are generally pulled away from the sea floor due to reflectivity of the sea floor. This makes the mean–estimate a poor choice for separating real targets from false alarms generated at the sea floor. The MAP–estimate has no obvious weaknesses in that regard, and is therefore used here.

The PPD is so far derived for single–ping information only. Assuming stationary target depth and independent measurements of arrival time and arrival angle, then the theory may be extended to apply for multi–ping information as follows:

\[
P(z_k|d(M)) = \frac{\prod_{j=0}^{N-1} P_{dz}(d(m_j)|z_k)}{\int_{z_{\text{min}}}^{\infty} \prod_{j=0}^{N-1} P_{dz}(d(m_j)|z)dz} \tag{13}
\]

\(m_j\) is the model parameter vector that applies for ping number \(j\). \(M\) is the model parameter matrix containing the model parameter vectors for all pings. The elements of the vector \(d(M)\) are target depths estimated by inputting \(m_j\) in (3). \(P_{dz}(d(m_j)|z_k)\) is determined from (9). The MAP–estimate then becomes:

\[
z_{MAP} = \max_{z_k} P_{zd}(z_k|d(M)) \tag{14}
\]

The theory may also be extended to limit the stationarity requirement for some or all model parameters. For instance, requiring that the sound speed profile remains the same for all pings, but allowing a non–stationary arrival time and arrival angle due to sonar and target movement. This extension is not within the scope of this work and therefore not included here.

### 3 Experimental data

A sea trial was conducted in the Norwegian trench at approximately 60° North and 4° East. The area is virtually flat with a sea floor depth of 300 m. The sound speed profiles were measured six times during the trial, see Fig. 2.

The vessel was equipped with a hull-mounted sonar, at 5 m depth, working on frequencies below 10 kHz. The sonar transmitted hyperbolic FM pulses with 2 kHz bandwidth and 1 s pulse length. The vertical beamwidth was 15°.

Several oil pipelines located on the seafloor, at approximately 300 m depth, crisscross the area of the sea trial. The sonar detected and maintained tracks on several pipelines. Measured arrival times and vertical arrival angles for one such track are shown in Fig. 3. This track was maintained for 16 pings.

An echo repeater was located in the area. The echo repeater was attached to a buoy and left to drift. The length of the cable from the buoy to the echo repeater was 81 m. The true
Figure 1: (a-d) show ray paths for four different pings. Each ray represents a single realisation of the random model input vector given by $m_j$. The colour of the ray represents the a priori probability of the model given by $P_m(m_j)$. The a priori probabilities of all rays with end points within a single depth interval, centered at $z_k$, are summed to find the probability, $P_{dz}(d(m)|z_k)$, that the modelled target depth lies in that depth interval. (e-h) show the PPD estimated using (8). (i) shows the PPD based on all four measurements and is essentially the normalised product of (e-h), see (13).

Figure 2: Measured sound speed profiles in the Norwegian trench at the time of the sea trial.

depth of the echo repeater is unknown, but is clearly less than 81 m. The sonar detected and maintained a track on the echo repeater for 87 pings. Measured arrival times and vertical arrival angles for this track are shown in Fig. 3.

4 Results and discussion

The described method is tested on data measured during the sea trial described in section 3. A raytracer called PlaneRay [17] is used. The model parameter vector, $m$, includes sound speed profile, sonar depth, arrival time, and vertical arrival angle. Each of these parameters are considered as Gaussian distributed random parameters.

On a flat sea surface, the sonar depth is 5 m below the sea surface. However, due to ship heave, the sonar depth is considered uniformly distributed between 2 m and 8 m. The vertical arrival angle is considered Gaussian distributed with a standard deviation given by (1). Angles within two standard deviations of the measured arrival angles are considered. The uncertainty in arrival time is so low that errors of several standard deviations in size have no significant impact on the results. The arrival time is therefore considered as a deterministic constant.

The sound speed profile is represented by empirical orthogonal functions (EOF). EOFs are derived from the measured sound speed profiles shown in Fig. 2. Details on how this is done can be found in [16]. The proportion of variances [16] for the two first EOFs are 85 %, which is assumed sufficient to describe the sound speed profiles. Sound speed profiles are generated as follows:

$$c(\kappa) = \bar{c} + U^T \kappa$$  \hspace{1cm} (15)

$\bar{c}$ is a vector containing the expected sound speed profile estimated by averaging the measured sound speed profiles. $U$ is a matrix containing EOFs as rows. $\kappa$ is a vector containing EOF coefficients. The EOF coefficients are zero–mean, Gaussian–distributed random processes and are included in the model parameter vector. The standard deviations of the coefficients are given by the square root of the corresponding eigenvalues [15]. Coefficient values within two standard deviations of 0 are considered.

Target depth PPDs based on the two tracks presented in section 3 are estimated using the described method. Ten depth–intervals are used in (9), each 30 m wide. Fig. 4 shows the target depth PPDs for the detected pipeline for all 16 pings.
The method estimates the target depth to be close to the bottom for all pings. Fig. 5 shows the PPD accumulated over all pings. The MAP–estimate for the target depth is 284 m, and the estimated standard deviation is 10 m. The true depth of the target is 300 m.

Fig. 6 shows the target depth PPDs for the echo repeater. The method estimates the target depth to be within the upper 60 m of the water–column for most pings. For a few pings, the PPD is almost uniform. These are pings where the SNR is low, resulting in high vertical angle standard deviations (1). Due to their high estimated target depth standard deviation, these pings have little influence on the PPD estimated from all pings shown in Fig. 7. The MAP–estimate for the target depth is 45 m, and the estimated standard deviation is 9 m. The true target depth is unknown, but below 81 m.

The accuracy of the target depth estimates is considered sufficient for classification purposes, since the main goal is to separate targets in the upper water–column from targets located on the sea floor.

5 Conclusion

A method capable of estimating the depth of a submerged target using mid–frequency, active sonar data is demonstrated. Ray backpropagation and Bayesian inversion are combined. The methods input is the present environment and recorded arrival times and vertical arrival angles.

The method is applied on a data set containing acoustic returns from a pipeline located on the sea floor at 300 m depth and an echo repeater located at an unknown depth less than 81 m. The a posteriori target depth probability density function was derived using Bayesian inversion and ray backpropagation. The sound speed profile was represented by empirical orthogonal functions (EOF), and the resulting EOF co-efficients were included in the inversion. Sonar depth was also varied in the inversion process. The maximum a posteriori (MAP) estimate of the depth of the pipeline was 285 m and the standard deviation was 10 m. The MAP estimate of the depth of the echo repeater was 45 m with a standard deviation of 9 m.

Knowing the depth of a target is a powerful tool for classification of submarines and mine–like objects in the ocean. Filtering tracks on account of target depth may reduce the false alarm rate in littoral areas significantly.

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Figure 7: The target depth PPD for the echo repeater estimated using (13) is plotted. This is essentially the product of all PPDs shown in Fig. 6.

References


Paper 5

Inverting the water column sound speed
Inverting the water column sound speed

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Sonar performance models are commonly used in anti-submarine warfare operations. The validity of sonar performance models are generally limited by available environmental information, such as the present sound speed profile. However, frequent sound speed measurements are costly and slows down the operation.

This work presents an inversion method for estimating sound speed profiles by exploiting available sensor information. Sensor information considered includes sound speed measurements close to sonar equipment, and echo sounder data used to estimate depth-averaged slowness. Predicted oceanographic data and a bottom model are required. The inversion method derives empirical orthogonal functions (EOF) for modelled sound speed profiles. EOF coefficients are varied using conjugate gradient searches, in order to estimate a sound speed profile that matches well with measured values collected from the sensors.

The method is applied on simulated data. The inverted sound speed profiles are shown to resemble the simulated truth data well, and that sonar performance modelling based on inverted sound speed profiles is comparable to performance modelled on basis of four-hourly sound speed measurements. Furthermore, inverted sound speed profiles are shown useful for quality assessment of sound speed measurements.

1 Introduction

Validity of sonar performance models is generally limited by environmental uncertainty. James and Dowling [1] give an overview of research on how environmental uncertainty influences acoustic field predictions. Modelled acoustic fields are sensitive to uncertain water-column sound speed [2–5]. Frequent sound speed measurements reduce the uncertainty, but may be costly and is not always feasible. An alternative approach is to invert the sound speed profile (SSP) from recorded acoustic data. Collins and Kuperman introduced focalisation [6] as a means of improving source localisation by including the SSP in matched-field processing. Empirical orthogonal functions (EOFs) [7] were used to characterise the SSPs. The EOF coefficients were used as additional parameters in the inversion process. Inverting SSPs from acoustic data in order to improve acoustic modelling, has since been used in matched-field processing [8, 9] as well as other methods [10, 11].

This work presents an inversion method that estimates a local SSP from available sensor information using inversion. Sensor information considered are sound speed measured close to sonar equipment, and echo sounder data. Echo sounder data are used to estimate depth-averaged slowness. The measurements are performed in the immediate vicinity of the sensors and have high-update rates, and are therefore considered local in space and time.

The inversion method is applied on a simulated data set. In the simulations, a set of SSPs collected during a sea trial in the Norwegian trench using a moving vessel profiler [12], are simulated as truth data. Measurements are simulated by extracting data at the measurement depth from the truth data. Predicted SSPs from the MI-POM ocean model [13] are used to determine a set of EOFs. EOF coefficients are varied using conjugate gradient searches [15] in order to generate an SSP matching the simulated measurements. Inverted SSPs are compared to truth data. Modelled sonar performance using inverted SSPs is compared to modelled sonar performance using sound speed measurements. The acoustic model Lybin [14] is used to model the sonar performance. A method of assessing the quality of sound speed measurements using inverted sound speed profiles is also presented and discussed.

2 Method

The proposed method employs an inversion approach to estimate the SSP from a limited set of sound speed measurements. The inverted SSP is generated using EOFs. EOF coefficients are varied in order to minimise the difference between measurements and the inverted SSP. For easier implementation, slowness, $s$, is used instead of sound speed, $c$:

\[ s = \frac{1}{c} \]  

Section 2.1 shows how EOFs are generated from a set of SSPs. Costfunctions for minimisation and a minimisation method are presented in sections 2.2 and 2.3. A function for comparing modelled sonar performance for two different input SSPs is described in section 2.4.
2.1 Determining the empirical orthogonal functions for a set of measured slowness profiles

Consider a set of \( M \) measured slowness profiles each containing \( N \) equally gridded depth samples. Let \( s_m \) be an \( N \)-dimensional vector containing the slowness values from the \( m \)th profile. Let the slowness data matrix, \( S \), be given by:

\[
S = \begin{bmatrix}
s_0^T \\
s_1^T \\
\vdots \\
s_{M-1}^T
\end{bmatrix}
\]  

(2)

The mean slowness, \( \bar{s} \), is given by:

\[
\bar{s} = \frac{1}{M} S^T v
\]  

(3)

\( v \) is an \( M \)-dimensional vector consisting of ones only. Let the scaled slowness, \( x \), be given by:

\[
x = s - \bar{s}
\]  

(4)

The scaled slowness data matrix is defined as:

\[
X = \begin{bmatrix}
x_0^T \\
x_1^T \\
\vdots \\
x_{M-1}^T
\end{bmatrix}
\]  

(5)

\( x_m \) may be expanded in any set of \( N \) orthonormal eigenvectors \( u_n \) with coefficient vector \( \kappa_m \), section 4.1 [7]:

\[
x_m = U^T \kappa_m
\]  

(6)

\( U \) is the eigenmatrix given by:

\[
U = \begin{bmatrix}
u_0^T \\
u_1^T \\
\vdots \\
u_{N-1}^T
\end{bmatrix}
\]  

(7)

The eigenvectors are found by solving the eigenvalue problem [7]:

\[
R_x u_n = \lambda_n u_n
\]  

(8)

In literature \( u_n \) are often referred to as empirical orthogonal functions (EOF). \( R_x \) is the covariance matrix of the scaled slowness data matrix [7]:

\[
R_x = \frac{1}{M} X^T X
\]  

(9)

\( \lambda_n \) is the eigenvalue corresponding to eigenvector \( u_n \). The eigenvalues equal the variances of the corresponding coefficients \( \kappa_n \) [7]:

\[
\text{Var}(\kappa_n) = \lambda_n
\]  

(10)

New slowness profiles of the same class as the slowness profiles in the original data set may be generated by using (4) and (6). For an arbitrary coefficient vector \( \kappa \), a new profile, \( \hat{s} \), may be generated using the following equation:

\[
\hat{s} = U^T \kappa
\]  

(11)

Random slowness profiles may be generated by modelling the coefficient vector as a zero–mean random vector with variances given by the corresponding eigenvalues.

Let \( \lambda_n \) be ordered such that for increasing \( n \), \( \lambda_n \) decreases. Higher orders of \( n \) often represent noise [7]. The vectors in (11) may therefore be truncated with insignificant loss of information on the slowness profile. A commonly used approach is to cut–off when the proportion of variances, \( \Gamma_I \), exceeds a selected threshold. The proportion of variances is defined as:

\[
\Gamma_I = \frac{\sum_{n=0}^{N} \lambda_n}{\sum_{n=0}^{N} \lambda_n}
\]  

(12)

The chosen threshold depends on the specific application, but a commonly used threshold is 95%.

2.2 Minimisation problem

Let the eigenmatrix, \( U \), and eigenvalue vector, \( \lambda \), be estimated from a set of \( M \) slowness profiles, as described in the previous section. Let \( d \) be an \( J \times K \) matrix containing \( K \) independent measurements made on \( J \) different sensors (e. g. depth–averaged slowness from an echo sounder or single–depth slowness values from sound speed sensors). If \( J = N \), the coefficients, \( \kappa \), may be found analytically. If \( J < N \), then the system of equations is indeterminate and may not be solved directly. However, the problem may be solved by inversion, where a set of coefficients is sought that results in a generated slowness profile, \( \hat{s}(\kappa) \), best matching the measured slowness values. Let \( \hat{d}(\kappa) \) be a vector containing values derived from \( \hat{s}(\kappa) \) that corresponds to the measurements \( d \).

\( d \) and \( \hat{d}(\kappa) \) are related by Bayes theorem [16]:

\[
P(\hat{d}(\kappa)|d)P(d) = P(d|\hat{d}(\kappa))P(\hat{d}(\kappa))
\]  

(13)

The measurements, \( d \), are regarded as fixed values, \( P(d) \) is therefore constant. \( \kappa \) is a random vector with independent elements and variance \( \lambda \). The prior coefficient probability is then given by:

\[
P(\kappa) = \prod_{n=0}^{N-1} P(\kappa_n)
\]  

(14)

\( \kappa \) is assumed Gaussian distributed. The probability density function relating to \( P(\kappa) \) is then given by:

\[
f_{\kappa}(\kappa) = \frac{1}{\sqrt{2\pi\lambda_n}} \exp \left( -\frac{\kappa_n^2}{2\lambda_n} \right)
\]  

(15)
Assuming a one-to-one relationship between \( \kappa \) and \( \hat{d}(\kappa) \) and since \( \kappa \) is the only random input to \( d(\kappa) \), then:

\[
P(d(\kappa)) = P(\kappa) \quad (16)
\]

\( P(d(\tilde{d}(\kappa))) \) is interpreted as the likelihood function, and the elements of \( d(\kappa) \) are assumed independent and Gaussian distributed. The likelihood function is then given by:

\[
L(\kappa) = \prod_{j=0}^{J-1} L_j(\kappa) \quad (17)
\]

where,

\[
L_j(\kappa) = \frac{1}{\sqrt{2\pi\sigma_{d_j}}} \exp \left( -\frac{1}{2} \sum_{k=0}^{K-1} \frac{(d_{jk} - \hat{d}_j(\kappa))^2}{\sigma_{d_j}^2} \right) \quad (18)
\]

\( d_{jk} \) are the elements of \( d \). \( \sigma_{d_j} \) is the estimated standard deviation of the \( j \)th sensor. \( \hat{d}_j(\kappa) \) are the elements of \( \hat{d}(\kappa) \). The marginal probability density function that relates to the probability \( P(d) \) may be determined by integrating the likelihood function over all \( \kappa \):

\[
f_d(d) = \int_{-\infty}^{\infty} L(\kappa)d\kappa \quad (19)
\]

Combining (13–19) gives the posterior probability density (PPD) for the EOF coefficients:

\[
f_{\hat{d}}(\hat{d}(\kappa))|d(\kappa)) = \prod_{j=0}^{J-1} L_j(\kappa) \prod_{n=0}^{N-1} f_{\kappa_n}(\kappa_n)/\int_{-\infty}^{\infty} L(\kappa)d\kappa \quad (20)
\]

The MAP–estimates of the coefficients are found at the maximum value of the PPD. The PPD maximum value is found by maximising the exponents in (15) and (18), which is equivalent to the following minimisation problem:

\[
\min_{\kappa \in \mathbb{R}^T} (F(\kappa)) = \min_{\kappa \in \mathbb{R}^T} \left( \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} \frac{(d_{jk} - \hat{d}_j(\kappa))^2}{\sigma_{d_j}^2} + \sum_{n=0}^{N-1} \frac{\kappa_n^2}{\lambda_n} \right) \quad (21)
\]

\( F(\kappa) \) is the cost function of the inversion problem, and the coefficient vector that minimises \( F(\kappa) \) yields the optimal inverted slowness profile. According to Dosso et al. [16] the standard deviations, \( \sigma_{d_j} \), may be estimated from the following expression:

\[
\sigma_{d_j}^2 = \frac{1}{K} \sum_{k=0}^{K-1} (d_{jk} - \hat{d}_j(\kappa))^2 \quad (22)
\]

where \( \hat{d}_j(\kappa) \) is estimated by solving the following minimisation problem:

\[
\min_{\kappa \in \mathbb{R}^T} (F(\kappa)) = \min_{\kappa \in \mathbb{R}^T} \left( \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} (d_{jk} - \hat{d}_j(\kappa))^2 + \sum_{n=0}^{N-1} \frac{\kappa_n^2}{\lambda_n} \right) \quad (23)
\]

\[
\sigma_{m_j}^2 = \frac{1}{\lambda_j} \sum_{k=0}^{K-1} (m_{jk} - \hat{m}_j(\kappa))^2 \quad (24)
\]

\( m_{jk} \) is the depth of the echo sounder. \( T_k \) is the \( k \)th measurement of the time it takes a transmitted acoustic pulse to reach the bottom and back again. \( D \) is the bottom depth. The depth–averaged slowness may be estimated from a generated slowness profile:

\[
\hat{m}_j(\kappa) = \frac{1}{N-n_j} \sum_{j=n_j}^{N-1} \hat{\kappa}_j \quad (25)
\]

\( \hat{m}_j(\kappa) \) are the elements of \( \hat{m}(\kappa) \).

\[
\min_{\kappa \in \mathbb{R}^T} (F(\kappa)) = \min_{\kappa \in \mathbb{R}^T} \left( \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} \frac{(d_{jk} - \hat{d}_j(\kappa))^2}{\sigma_{d_j}^2} + \sum_{n=0}^{N-1} \frac{\kappa_n^2}{\lambda_n} \right) \quad (26)
\]

\( m_{jk} \) and \( m_{jk}^* \) are scaled versions of the measured slownesses and depth–averaged slownesses, respectively. The costfunction may be split in three:

\[
F(\kappa) = F_s(\kappa) + F_m(\kappa) + F_e(\kappa) \quad (27)
\]

\[
F_s(\kappa) = \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} \frac{(x_{jk} - \hat{x}_j(\kappa))^2}{\sigma_j^2} \quad (28)
\]

\[
F_m(\kappa) = \frac{1}{\lambda_j} \sum_{k=0}^{K-1} (m_{jk} - \hat{m}_j(\kappa))^2 \quad (29)
\]

\[
F_e(\kappa) = \sum_{n=0}^{N-1} \frac{\kappa_n^2}{\lambda_n} \quad (30)
\]
Inserting (11) and (25):

\[
F_x(\kappa) = \sum_{j=0}^{N-1} \sum_{k=0}^{K-1} \frac{d_j}{\sigma_{xj}} (x_{jk} - \sum_{n=0}^{N-1} U_{jn} \kappa_n)^2 \quad (32)
\]

\[
F_m(\kappa) = \frac{1}{\sigma_{mn}} \sum_{k=0}^{K-1} \left( \sum_{n=0}^{N-1} U_{jn} \kappa_n \right)^2 \quad (33)
\]

Equations (31–33) are all differentiable with respect to \( \kappa_n \):

\[
\frac{\partial F_x(\kappa)}{\partial \kappa_n} = -2 \sum_{j=0}^{N-1} \sum_{k=0}^{K-1} a_j U_{jn} \sigma_{xj}^{-2} (x_j - \sum_{n=0}^{N-1} U_{jn} \kappa_n)
\]

\[
\frac{\partial F_m(\kappa)}{\partial \kappa_n} = -\frac{2}{N - n_x} \sum_{j=0}^{N-1} U_{jn} \sigma_{mn}^{-2} \sum_{k=0}^{K-1} \left( m_{sk} - \sum_{n=0}^{N-1} U_{jn} \kappa_n \right)
\]

\[
\frac{\partial F_x(\kappa)}{\partial \kappa_n} = 2 \sum_{n=0}^{N-1} \kappa_n \quad (34)
\]

The cost function differentiated with respect to \( \kappa_n \) then becomes:

\[
\frac{\partial F(\kappa)}{\partial \kappa_n} = \frac{\partial F_x(\kappa)}{\partial \kappa_n} + \frac{\partial F_m(\kappa)}{\partial \kappa_n} + \frac{\partial F_k(\kappa)}{\partial \kappa_n} \quad (34)
\]

Since the cost function and its derivatives are analytically known, the minimisation problem in (26) may be solved using a conjugate gradient method [15]. The computational cost may be strongly reduced by first truncating the coefficient vector with insignificant loss of accuracy, as discussed in section 2.1.

### 2.4 Comparing modelled sonar performance

In anti–submarine warfare operations sonar performance modelling is mainly used for determining whether the submarine is detected or not. Let the detection matrix, \( \mathbf{D} \), be given by:

\[
\mathbf{D}(\mathbf{m}) = H(\mathbf{SE}(\mathbf{m}))
\]

(35)

The modelled signal excess [17], \( \mathbf{SE} \), is a matrix containing logarithmic signal excess values for all model depth– and range–cells. The depths and ranges are typically bounded by the sonar range, surface, and bottom depth. \( \mathbf{m} \) is the model input, such as environment and sonar parameters. \( H(x) \) is the Heaviside–function, which is defined as 1 for \( x \geq 0 \) and 0 otherwise. The detection matrix shows at what depths and ranges the sonar is expected to detect a submarine.

Let the model inputs \( \mathbf{m}_1 \) and \( \mathbf{m}_2 \) contain different SSPs, but be otherwise identical. Let the SSPs be given by \( \mathbf{c}_1 \) and \( \mathbf{c}_2 \). Let the compare function, \( G(\mathbf{c}_1, \mathbf{c}_2) \), be defined as:

\[
G(\mathbf{c}_1, \mathbf{c}_2) = \frac{1}{N_r N_c} \sum_{m=0}^{N_r-1} \sum_{n=0}^{N_c-1} \delta(D_{mn}(\mathbf{m}_1) - D_{mn}(\mathbf{m}_2))
\]

(36)

\( N_r \) and \( N_c \) are the number of model range and depth cells, respectively. \( \delta(l) \) is the Kronecker–delta function and defined as 1 when \( l = 0 \) and 0 otherwise. \( D_{mn} \) are the elements of

D. The compare function is used to evaluate how well the modelled sonar performance matches for two different input SSPs. A value of 1 indicates that the detection functions are identical, while 0 indicates a complete misfit.

### 3 Environment

In January 2010 the Norwegian Defence Research Establishment (FFI) conducted a sea trial in the Norwegian trench. FFI’s research vessel, HU Sverdrup II, measured pressure, conductivity, and temperature as a function of depth down to approximately 200 m depth using an ODIM MVP200, a moving vessel profiler (MVP) [12]. SSPs are estimated from the measurements using the UNESCO formula [18]. Estimated SSPs (centre plot) and the path of the vessel (upper plot) are shown in Fig. 1.

An oceanographic model is available for the area the sea trial was conducted in. The model covers a 16 500 square kilometer area at the Western coast of Norway. The ocean model Westcoast-200m is a version of Princeton Ocean Model (POM) called MI-POM [13]. Fig. 2 shows SSPs extracted from the model for the area and time the sea trial was conducted in.

### 4 Simulated example

Consider a simulated sea trial where a sonar vessel follows the path shown in the upper plot in Fig. 1. Let the vessel’s position be given by \( \mathbf{x}_t \) at time \( t \). The SSPs shown in the centre plot in Fig. 1 simulate the true SSPs, \( \tilde{\mathbf{c}}_{t,j}^{(f)} \), at position \( \mathbf{x}_t \) at time \( t \). The true SSPs are assumed constant and range–independent at times close to \( t \) and positions close to \( \mathbf{x}_t \). \( j \) is the depth numerator. The depth is divided into 5 m intervals from 0 m to 200 m.

![Figure 1: Path of the vessel (upper plot). Measured SSPs (centre plot). Inverted SSPs (bottom plot). The numbers (time indexes) and stars along the path of the vessel indicate the positions where the sound speed is measured in the first three scenarios described in section 5.2.](image-url)
The simulated vessel is equipped with an echo sounder, a towed transmitter, and a towed receiver array. The echo sounder and the towed array are both equipped with sensors measuring the sound speed directly with standard deviation, \(\sigma_c\), of 0.2 \(\text{m}^{-1}\), e.g. an AQSv--1500 [22]. The echo sounder is mounted on the hull of the vessel at 5 m depth, corresponding to \(j = 1\). The towed receiver array and transmitter are kept at 100 m depth, corresponding to \(j = 20\). For each \(t_l\), 30 direct sound speed measurements and echo sounder pings are simulated. The sound speed measurements are simulated by adding noise to the simulated true sound speed at the corresponding position and measurement depth:

\[
\begin{align*}
c^{(l)}_{1k} &= c^{(l)}_1 + g \\
c^{(l)}_{20k} &= c^{(l)}_{20} + g
\end{align*}
\]

\(k\) is the measurement number. The noise is modelled as zero--mean, Gaussian distributed noise with standard deviation given by \(\sigma_c\).

Let the expected bottom depth be 200 m with a standard deviation of 0.5 m. Measured depth--averaged slowness is simulated by computing mean slowness from simulated true SSP at that position and adding a zero--mean, Gaussian distributed noise, \(h\), with standard deviation given by \(\sigma_m\):

\[
m^{(l)}_k = \frac{1}{N-1} \sum_{j=n_s}^{N-1} c^{(l)}_j + h
\]

\(n_s\) is the depth--sample corresponding to the echo sounder depth. \(N - 1\) is the depth--sample number corresponding to bottom depth. The standard deviation, \(\sigma_m\), is \(2 \cdot 10^{-6}\ \text{m}\), which corresponds to 0.5 m standard deviation in the bottom model.

EOFs are derived from the SSPs in the oceanographic model, shown in Fig. 2. The four first coefficients are used in the inversion, resulting in a proportion of variances of 0.98. A single inverted SSP, \(\tilde{c}^{(l)}\), is determined for each \(t_l\) by solving (21). The MATLAB function MINIMIZE.M created by Carl Edward Rasmussen [19, 20] is used. It utilizes conjugate gradients and approximate linesearches. All inverted SSPs are shown in the lower plot in Fig. 1. The sound speed is generally underestimated in the lower 100 m, while the upper half of the SSPs are visually quite convincing. This is as expected since the two direct sound speed measurements made are in the upper half of the water--column; 5 m and 100 m depths.

5 Applications

Two applications using inverted SSPs are discussed on basis of the simulated data in the previous section: Quality assessment of measured SSPs and sonar performance modelling using inverted SSPs.

5.1 Quality assessment of measured sound speed profiles

This section shows how inverted SSPs may be used to assess the quality of measured SSPs. Let the vessel in the simulated example in section 4 make a single SSP measurement, given by \(c^{(m)}\), at time \(t_m\) during the sea trial. The SSP measurement is simulated as follows:

\[
c^{(m)} = \tilde{c}^{(m)}
\]

Let all true SSPs have an equal probability of being the one measured. At time \(t_l\) the quality, \(Q_{lm}\), of the measured SSP is given by:

\[
Q_{lm} = G(\tilde{c}^{(l)}, c^{(m)})
\]

where \(G\) is defined in (36). In a real sea trial this is an unknown quantity since the true SSP is unknown. However, \(\hat{Q}_{lm}\) may be modelled by using the inverted SSP at time \(t_l\) instead of the true SSP:

\[
\hat{Q}_{lm} = G(\tilde{c}^{(l)}, \tilde{c}^{(m)})
\]

Fig. 3 shows \(\hat{Q}_{lm}\) plotted versus \(Q_{lm}\) for all combinations of \(l\) and \(m\). Notice that the main bulk of instances result in values close to 1.

The quality assessment of the measured SSP may be formulated as a binary hypothesis test. For each time \(t_l\) there are two outcomes of the test:

1. The measured SSP is high--quality
2. The measured SSP is low--quality

This is a binary decision problem [21], where the goal is to detect and report low--quality SSPs. Quality is determined by applying a threshold, \(T_0\), on \(Q_{lm}\):

\[
Q_{lm} \geq T_0 \Rightarrow \text{high--quality SSP} \\
Q_{lm} < T_0 \Rightarrow \text{low--quality SSP}
\]
$T_0$ is here chosen to be 50%, but should be sufficiently high to meet any requirements on the quality of the sonar performance modelling. Since $Q_{lm}$ is unknown, this metric may not be used in the assessment. Instead the known quantity $\hat{Q}_{lm}$ may be used:

\[
\hat{Q}_{lm} \geq T \Rightarrow \text{high–quality SSP predicted}
\]
\[
\hat{Q}_{lm} < T \Rightarrow \text{low–quality SSP predicted}
\]

The threshold $T$ is here chosen to be 0.5 as indicated in Fig. 3. The test may result in four different outcomes:

1. The measured SSP is high–quality
   a) high–quality predicted
   b) low–quality predicted

2. The measured SSP is low–quality
   a) high–quality predicted
   b) low–quality predicted

Alternatives 1a and 2b are correct decisions, and correspond to the upper right and lower left quadrants in Fig. 3, respectively. 1b is a false alarm and 2a is a failed detection, corresponding to the lower right and upper left quadrants of Fig. 3, respectively.

Let probability of detection, $P_d$, be the probability that a low–quality profile is chosen to be low–quality by the test, and probability of false alarm, $P_{fa}$, be the probability that a high–quality profile is chosen to be low–quality. $P_d$ and $P_{fa}$ may be estimated from the simulated data set as follows:

\[
P_d(T) = \frac{\sum_l \sum_m H(T - \hat{Q}_{lm})H(T_0 - Q_{lm})}{\sum_l \sum_m H(T_0 - Q_{lm})}
\]
\[
P_{fa}(T) = \frac{\sum_l \sum_m H(T - \hat{Q}_{lm})H(Q_{lm} - T_0)}{\sum_l \sum_m H(Q_{lm} - T_0)}
\]

where $H(x)$ is the Heaviside function. By varying $T$ from 0 to 1 in 0.1 steps receiver operating characteristic (ROC) curves [21] are estimated and shown in Fig. 4. The proposed detector algorithm performs significantly better than random decision. For $T = 0.5$ and $T_0 = 0.5$ there is a 61% probability of detecting a low–quality SSP and 6% probability of false alarm. The false alarm rate may be suppressed further by for instance requiring two consecutive detections before reporting a SSP as low–quality, all though this would decrease the probability of detecting a low–quality SSP as well.

5.2 Sonar performance modelling

In this section the quality of modelled sonar performance using inverted SSPs is compared to the quality of modelled sonar performance using SSP measurements of varying frequency.
The quality of each SSP in each scenario is found by comparing the profiles to the true SSPs using the method described in section 2.4:

\[ Q_1^{(l)} = G(c_1^{(l)}, c_1) \]
\[ Q_2^{(l)} = G(c_1^{(l)}, c_2^{(l)}) \]
\[ Q_3^{(l)} = G(c_1^{(l)}, c_3^{(l)}) \]
\[ Q_4^{(l)} = G(c_1^{(l)}, c_4^{(l)}) \]

\( c_1 \) equals the measurement in the first scenario, as described above. \( c_2^{(l)} \) and \( c_3^{(l)} \) equal the most recent measurements in the second and third scenarios, respectively. The results of the comparison are presented in Fig. 5. The averaged quality over all timesteps are 0.73 in the first scenario, 0.77 in the second, and 0.79 in the third scenario. When using the inverted SSPs, the averaged quality is 0.78. For simple sonar performance modelling, where the main goal is to determine the maximum detection range of a sonar, the inverted SSPs have a quality that is comparable to four-hourly sound speed measurements.

\section{Conclusion}

A new method for inverting water-column sound speed profiles is presented. The inversion is based on sound speed and echo sounder measurements made from the sonar platform. The high update-rates of these sensors ensure a local sound speed profile.

Sound speed profiles were inverted using a simulated data set. The inverted sound speed profiles resembled the true sound speed profiles. However the method must be tested on recorded data for final validation.

An algorithm for detecting poor-quality sound speed profiles was developed and tested on the simulated data set. Inverted sound speed profiles were used to assess the quality of measured sound speed profiles. By sound speed profile quality is here meant how well the modelled sonar performance using that particular sound speed profile compare to the modelled sonar performance using the true sound speed profile. The algorithm was shown to have a probability of detection of 61\% and a probability of false alarm of 6\% for the simulated test case.

The inverted sound speed profiles were also shown to result in modelled sonar performance comparable to four-hourly measured sound speed profiles for the simulated test case. The inversion method gives some operational advantages during an actual anti-submarine warfare operation. The sound speed profile may be inverted for any vessel speed, unlike the traditional expendable sound velocity profiler which has an upper speed limit, e.g. 8 m/s for a Sippican XSV-01 [23]. Secondly, the update rate of the inverted sound speed profile is higher than any other sound speed profiler known to the author. The measurement frequency depends on the bottom depth, due to the echo sounder, while the computation time of the method is a few seconds on a standard desktop computer.

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\section{References}


Paper 6

Finding acoustically stable areas through EOF classification
Finding acoustically stable areas through EOF classification

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Abstract

Validity of sonar performance models is generally limited by environmental uncertainty, and particularly uncertainty in the sound speed profile (SSP). Rapid environmental assessment (REA) missions, e. g. using gliders, and advanced ocean models may be used to reduce this uncertainty prior to sonar operation in hostile waters.

The presented work shows how data from ocean models may be used for planning of REA–missions. The area of operation is divided into oceanographically stable subareas using empirical orthogonal functions and different methods of clustering analyses on SSPs from the ocean model. The acoustic stability of each subarea is assessed using sonar performance modelling. Acoustically unstable areas are divided into smaller subareas. Acoustically stable groups are represented by a single SSP.

A map of acoustically stable areas in the area of operation is the main output. Large, geographically contiguous groups indicate acoustically stable areas where frequent SSP measurements are unnecessary, e. g. low concentration of gliders. Small and non–contiguous groups indicate the opposite. Other applications include determination of suitable locations for sonar tests that require stable sonar conditions, and efficient optimization of sonar operation in acoustically stable areas.

I. INTRODUCTION

Validity of sonar performance models is generally limited by environmental uncertainty. James and Dowling [1] give an extensive overview of research on how environmental uncertainty influences acoustic field predictions. Modelled acoustic fields are sensitive to uncertainty in sound speed profile (SSP) [2]–[7]. During sonar operations, sonar performance models are used for prediction of sonar ranges and for determining optimal placement of sonar assets during multi–platform sonar operations. A consequence of uncertainty in predicted sonar range is that more sonar assets are required during a sonar operation to obtain the required probability of success. This uncertainty can be reduced by increasing the knowledge of the environment, for instance by using ocean models or conducting rapid environmental assessment (REA) missions in the area of operation, e. g. glider operations.

Davis [8] introduced empirical orthogonal functions (EOF) to characterise oceanographic data. EOF has since been extensively used to describe and characterise water masses and spatial and temporal variations of the ocean.
e. g. [9]. EOFs are also frequently used in acoustic inversion and tomography e. g. [10], [11]. The main advantage of characterising sound speed profiles by EOFs is that information from the entire SSP may be contained in a few scalar coefficients. For example, two coefficients are sufficient to characterise the Munk–profile [11].

Clustering analysis is a multivariate statistical technique for grouping data points with similar characteristics into clusters. In oceanography, it has most commonly been used to label and identify water masses according to measurements of depth, temperature and salinity [12]–[14]. In this study we take the leading coefficients from the EOF analysis of SSP as input to our cluster analysis. This is a method frequently used on echo sounder data for sea bottom classification e. g. [15]. The EOF analysis reduces the dimensionality of the considered data set before the cluster analysis classifies the data set into several different categories. This has the desired effect of clustering SSPs with similar acoustic properties. A sonar performance model is then used to assess the acoustic sensitivity to oceanographic variations within those groups. This proves to be a useful tool for quick classification of acoustically stable versus unstable groups of SSPs. The main output of the method is a map of acoustically stable areas, where an acoustically stable area contains SSPs from the same group. Such maps are useful planning aids during REA missions and sonar operations since they indicate the presence of stable and unstable regions. Stable regions are typically dominated by large, geographically contiguous stable areas. Frequent SSP measurements are unnecessary in such regions, e. g. low concentration of gliders. Unstable regions typically consists of many small and non-contiguous stable areas. In such areas frequent SSP measurements are required. The information is also useful for determining a suitable area for conducting sonar tests that require stable acoustic conditions. Finally, a single SSP is assumed sufficient to model a representative sonar performance for an acoustically stable area. Optimal sonar parameters for sonar operation in each area may then be obtained with low computation cost, since the sonar performance is modelled for a single SSP only. Examples of how optimal sonar performance is determined using acoustic modelling can be found in literature, e. g. [16].

The method is tested on SSPs from MI-POM [17], a high resolution numerical ocean model covering 16.000 square kilometers adjacent to the Norwegian West coast. Using this high resolution model as a basis for our calculations gives us a gridded data set where oceanographic dynamical features are realistically resolved. From an acoustic point of view, the main interest lies in the horizontal and vertical gradients of sound speed that is associated with the interface between low temperature, low salinity coastal water masses and comparatively warmer and more saline water masses of Atlantic origin. EOFs are determined for the modelled SSPs, and the geographical distribution of the first EOF coefficient is shown to correlate well with the distribution of upper layer salinity. The full method is applied on the data set and a map of acoustically stable areas is presented.

II. Method

The presented method divides a large set of SSPs into several smaller groups of profiles that are acoustically stable. A group is considered acoustically stable if variations in modelled signal excess [18], using the sound speed profiles of that group, are lower than a chosen threshold (section II-C). Groups not acoustically stable are split into smaller groups using a subdivision algorithm (section II-B). The subdivision is based on properties of the
coefficients in an EOF representation of the SSPs (section II-A). The end product is a map showing the geographic extent of the acoustically stable groups.

The overall method is as follows:

1) A group/subgroup of SSPs are input
   a) The acoustic fitness of the inputted group/subgroup is determined
      i) Groups/Subgroups with acoustic fitness exceeding the threshold $T$ are accepted and not further processed.
      ii) Group/Subgroups with acoustic fitness lower than the threshold $T$ are passed on to step 2).

2) A subdivision algorithm splits the group/subgroup into two or more subgroups
   a) Subgroups with less than $K$ SSPs are removed
   b) Subgroups with more than $K$ SSPs are returned to step 1)

For step 2) two different subdivision algorithms are used: Clustering of coefficients (CC), described in section II-B1, and ordering of coefficient magnitudes (OCM), described in section II-B2.

A. Empirical orthogonal functions

Consider a set of $M$ SSPs each containing $N$ depth samples. Let $c_m(z_n)$ be the sound speed at depth $z_n$ in the $m$'th profile. The mean sound speed, $\bar{c}$, as a function of depth, is given by:

$$
\bar{c}(z_n) = \frac{1}{M} \sum_{m=0}^{M-1} c_m(z_n)
$$

Let $x_m[n]$ be defined as:

$$
x_m[n] = c_m(z_n) - \bar{c}(z_n)
$$

$x_m[n]$ may be expanded in any set of $N$ orthonormal basis functions $u_k[n]$ with coefficients $\kappa_k$, [19] chapter 4.6.1:

$$
x_m[n] = \sum_{k=1}^{N-1} \kappa_k^{(m)} u_k[n]
$$

For real values of $x_m[n]$ the coefficients for expanding $x_m[n]$ are given by:

$$
\kappa_k^{(m)} = \sum_{n=0}^{N-1} u_k[n] x_m[n]
$$

The basis functions are determined by solving the eigenvalue problem given by:

$$
R_x u_k = \lambda_k u_k
$$

where $u_k$ are eigenvectors and contain the function values of the discrete orthonormal basis functions $u_k[n]$. In literature $u_k[n]$ are often referred to as empirical orthogonal functions (EOF), $\lambda_k$ are the corresponding eigenvalues and equal the variance of the corresponding coefficients $\kappa_k$ [19]. $R_x$ is the covariance matrix of the data matrix:

$$
R_x = \frac{1}{M} X^T X
$$
where,

\[
X = \begin{bmatrix}
  x_1^T \\
  x_2^T \\
  \vdots \\
  x_M^T
\end{bmatrix}
\]  

(7)

New SSPs, \( c_i[z_n] \) may be generated by combining (2) and (3):

\[
c_i[z_n] = \overline{c}[z_n] + \sum_{k=1}^{N-1} \kappa_k u_k[n]
\]

(8)

The coefficients, \( \kappa_k \), should be modelled as zero-mean random processes with variance \( \lambda_k \).

Higher orders of \( k \) often represent noise in the measurements. The series in (8) may therefore be truncated without risking loss of information on the SSP. A frequently used approach to select the cut–off is to determine where the proportion of variances exceeds a set threshold. The proportion of variances is defined as:

\[
\Lambda_l = \frac{\sum_{k=0}^{l} \lambda_k}{\sum_{k=0}^{N-1} \lambda_k}
\]

(9)

\( l \) is the cut–off. The selected threshold depends on the applications, but a commonly used threshold is 95%.

B. Subdivision algorithms

1) Clustering of EOF coefficients (CC): Cluster analysis is assigning data points with similar characteristics to the same group of data, called cluster [20]. A common implementation, called hierarchical clustering, is to start with each data point contained within its own unique cluster, one point per cluster. Some function gives an objective measure of the similarity – or distance – between cluster pairs, and the most similar (closest) cluster pair is joined. The process is repeated until some metric of success is achieved (e.g. [21]) or when the process is subjectively judged to give the most meaningful representation of the data distribution. Among the vast multitude of possible functions, the Euclidean distance function is a straightforward choice with intuitive parallels to conventional geometry. Also, some choices must be made about defining distances between clusters with more than one member, such as closest neighbor, average midpoint, weighted average and so on.

In oceanography, cluster analysis has proven successful for identifying water masses according to the data points of temperature, salinity and depth [13], [14]. In this study we perform cluster analysis on the leading EOF coefficients for SSPs calculated in section II-A. SSP’s with similar vertical structure will have similar coefficients and be close together in coefficient space — and will be assigned to the same cluster. This method is strikingly similar to the bottom type classification of acoustic survey data by Milligan and others [22], but we have not been able to find literature where this is done for SSP’s or other oceanographic parameters.

The cluster analysis is merely a tool for the overall algorithm presented in II. Conceptually, we describe the overall method as starting with one large cluster which then is tested by the acoustic fitness function II-C. If approved, all is good; if it fails, then that cluster is split in two and the test is repeated for each of the new clusters.
Fig. 1. a) The area is divided into subgroups based on the absolute value of the strongest coefficients to the SSPs, in this case the first and second coefficient are the strongest, noted 1 and 2. b) Groups found in a) is split based on the sign of the strongest coefficient. Subgroups containing fewer than $K$ SSPs are removed and the acoustic stability of each remaining group is determined. c) Step a) is repeated for the unstable areas in b) using the second strongest coefficient. d) Step b) is repeated for the new areas found in c).

2) Ordering of EOF coefficient magnitude (OCM): Ordering of EOF coefficient magnitude is a method introduced here for dividing a large group of SSPs into several smaller subgroups with similar statistical properties. The main advantage of this method compared to previously described clustering method is its ability to process very large data sets. The full method, as described in section II, using the OCM subdivision algorithm, is illustrated with an example in Fig. 1.

The coefficient with the highest absolute value is found for each SSP and is henceforth called the strongest coefficient. The SSPs are divided into subgroups represented by their strongest coefficient. In this case, for all SSPs, the first or second coefficient is strongest and the areas containing each subgroup are represented by 1 and 2 in the plot (Fig 1 a). b) Each of the subgroups are split in two by taking the sign of the coefficient into account, which results in one subgroup with a positive strongest coefficient, and a subgroup with a negative strongest coefficient. Subgroups containing fewer than $K$ SSPs are removed, in this case the area with coefficient number 3 (both
positive and negative) as the strongest coefficient (Fig 1 b). The acoustic stability of each subgroup is tested. Stable subgroups are kept. Unstable subgroups are divided again by repeating the steps above on the second strongest coefficient (Fig 1 c). Two of the subgroups (1 and 2) in Fig 1 b) were found unstable and divided into smaller subgroups. For the SSPs in subgroup 1, the second and third coefficients are the second strongest coefficients. For subgroup 2 the first and third coefficients are second strongest, see Fig 1 c). These areas are again divided based on the sign of the second strongest coefficient. Note that subgroups 2 3 and 1 3 are removed as they contain less than $K$ SSPs. In Fig 1 d) the remaining areas are checked if they are acoustic stable. This process continues until all subgroups are found stable or too small.

C. Acoustic fitness function

Assume cylindrical symmetry and let the random function $SE(r, z, c)$ represent the true signal excess of a target located in $(r, z)$, where the vector $c$ contains the depth–dependent SSP. The SSP is assumed range–independent. Let $c$ be the only random parameter influencing $SE(r, z, c)$.

Given $N$ SSPs, $c_n$, where $n = 1, 2, 3, ..., N$, in an area, let $c$ for that area be uniformly distributed over the $N$ SSPs, meaning all SSPs have an equal probability of being the true SSP in the given area. $SE(r, z, c_n)$ is the modelled signal excess in dB using the $n$th SSP as input. $s(r, z, c_n)$ is the linear signal excess:

$$SE(r, z, c_n) = 10 \log_{10} s(r, z, c_n)$$ (10)

The expected signal excess in the area is estimated as the mean modelled, linear signal excess:

$$m_s(r, z) = \frac{1}{N} \sum_{n=0}^{N-1} s(r, z, c_n)$$ (11)

Let $P_{SE}(r, z, T_{\Delta SE})$ be the probability that the mean signal excess lies within $T_{\Delta SE}$ of the true signal excess at an arbitrary target location given by $(r, z)$:

$$P_{SE}(r, z, c, T_{\Delta SE}) = Pr \{ |10 \log_{10} m_s(r, z) - SE(r, z, c)| \leq T_{\Delta SE} \}$$ (12)

The true SSP, $c$, is unknown, but since all SSPs have an equal probability of being the true SSP, then (12) may be estimated as follows:

$$P_{SE}(r, z, T_{\Delta SE}) \approx \frac{1}{N} \sum_{n=0}^{N-1} H \left( 10 \log_{10} \left( \frac{m_s(r, z)}{s(r, z, c_n)} \right) - T_{\Delta SE} \right)$$ (13)

where $H(x)$ is the Heaviside function and outputs 0 for arguments lower than 0, and 1 otherwise.

$\hat{P}(T_{\Delta SE})$ is the probability that the modelled signal excess lies within $T_{\Delta SE}$ of the true signal excess at the target location. An a priori probability density distribution of the target location, $g(r, z, \phi)$, is required. If unknown, a uniform distribution is used. $P(T_{\Delta SE})$ is then given by:

$$\hat{P}(T_{\Delta SE}) = \int_0^{2\pi} \int_0^H \int_0^R P_{SE}(r, z, T_{\Delta SE}) g(r, z, \phi) r dr dz d\phi$$ (14)

$H$ is the bottom depth and $R$ is the sonar range.
The depths and ranges considered should be limited to regions with sufficient acoustic energy. This is implemented by adding the following constraint:

\[ 10 \log_{10} m_s(r, z) > T_{SE} \]  

This constraint is added to (14) yielding the acoustic fitness function (AFF):

\[ P(T_{\Delta SE}) = \int_0^{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_{SE}(r, z, T_{\Delta SE}) g(r, z, \phi) H(10 \log_{10} m_s(r, z) - T_{SE}) r dr dz d\phi \]  

AFF is used to determine the acoustic stability of a group of SSPs, as described in step 1 in section II.

A potential problem for large, acoustically stable groups, is that the group may contain several disparate sets of SSPs that would be best represented as different acoustically stable groups. This may occur because the number of SSPs in the largest of these sets is large enough to satisfy the requirement on \( P(T_{\Delta SE}) \), masking the presence of smaller but distinctly different sets of SSPs. Therefore we have added an additional requirement on acoustically stable groups: if the number of SSPs with negative argument in the Heaviside function in (13) exceeds a selected number, for instance 10% of the number of SSPs in the original data set, then the group is considered acoustically unstable.

The proposed fitness function is a robust means of assessing acoustic stability. Note that signal excess is estimated from incoherent transmission loss, acoustic phase is therefore completely ignored. The chosen fitness is useful for the application considered; sonar performance modelling. Other applications may require different fitness functions that may also include phase information, but this is not considered within the scope of this work.

III. DATA SETS

The SSPs used in this study are based on three dimensional forecasts of temperature and salinity for 12 UTC, March 7th, 2007 from the high resolution numerical ocean model Westcoast 200m. This ocean model is a version of Princeton Ocean Model (POM) called MI-POM [17], [23], run operationally by the Norwegian Meteorological Institute (met.no). The model domain covers an area of approximately 16,500 km², from 59.30 N 4 E to 61 N 5.75 E with a horizontal resolution of 200 m, see Fig. 2. The data are downsampled to a horizontal resolution of approximately 1 km, which we here refer to as the full data set. SSPs from surface to 200 m depth at nine depth levels are used and shown in Fig. 3. Most data from fjords and inlets are removed and locations where model depth is less than 200 m are excluded. The data set then totals 10033 profiles, from which 3 additional subsets are extracted: One subsampled to 2873 profiles in a semi–regular grid (blue outline in figure 2) and two smaller area subsets with resolutions identical to the full data set. Subset 1 is bounded by the coordinates 60.00–60.17 N, 4.00–4.51 E (720 profiles) and subset 2 is bounded by 60.47-60.92 N, 3.98-4.48 E (175 profiles), shown in Fig. 2.

IV. EXPERIMENTAL RESULTS

The two following subsections present results from an oceanographic analysis and results from the method described in section II. The oceanographic analysis compares surface salinity contours to geographic distributions of EOF coefficients. EOF coefficients for each SSP in the full data set and each subset are derived using (4). Fig.
4 shows the proportion of variances. The proportion of variances exceeds 95 % when using three coefficients for the full area data sets (full and reduced resolution), and five and four coefficients, respectively, for subset 1 and 2. The subdivision algorithms, CC and OCM use five and four coefficients, respectively.

**A. Investigation of the acoustic fitness function**

The proposed method divides a large group of SSPs into acoustically stable sub–groups using EOF and cluster analysis on the SSPs. For this to be meaningful acoustic stability must in some way correlate with SSP variability. This correlation is demonstrated in Fig. 5, where the AFF proposed in section II-C is shown to decrease for increasing sound speed variability.

The number of SSPs needed to ensure a tolerable accuracy in the AFF is studied in Fig. 6. The estimated standard deviation of \( P(T_{\Delta SE}) \) drops for increasing number of SSPs used to estimate \( P(T_{\Delta SE}) \). Sufficiently high \( K \) in step 2, see II, should be selected in order to ensure the desired accuracy in \( P(T_{\Delta SE}) \).
B. Oceanographic analysis

The study area is in the Norwegian Trench, close to the western coast of Norway. This part of the Norwegian Trench is relatively flat, with a depth around 300 meters, with a steep rise at the Norwegian Coast to the east. Further west, adjacent to the study area, is the slope leading up to the Norwegian Sea plateau, with a depth of about 100-150 meters. Circulation in the area is dominated by inflow and recirculation of saline Atlantic water (AW) which enters the North Sea from the north and the low salinity coastal waters (CW) in the Norwegian Coastal Current (NCC). The NCC originates in Skagerrak as a mixture of very low salinity water from the Baltic and water masses of Atlantic origin that has been more or less diluted through their residence in the North Sea [24]–[26]. The NCC follows the Norwegian coast, but with variable lateral extent which is mainly controlled by wind forcing and meander/eddy formation processes. The usual arsenals of frontal dynamics (frontal structures, filaments, meanders and eddies) are found at the transition between NCC and AW water masses [25], [27], [28].

In general water masses with salinity greater than 35 psu are referred to as AW and below 35 psu as CW, but...
34.5 psu is a better separation along most of the Norwegian coast [29]. In March, there is a strong correlation between water temperature and salinity, with CW being cold and fresh and AW being higher in temperature and salinity. Both our own observations as well as temperature and salinity values from the literature indicate that the typical difference in sound speed between the AW and CW water masses for March is on the order of 10 m/s. The typical distribution of the two water masses is that the CW is wedged between the Norwegian coast and the adjacent AW, the deepest part of CW being close to the coast. The upper layer salinity values is therefore a good indicator of the geographical distribution of the different water masses [29]. Fig. 7 shows surface salinity contours and the first EOF coefficient plotted geographically. Observe that most AW areas is associated with negative values of the first coefficient and most of the positive values correspond to CW. Since 80% of the variance is contained in the first coefficient, see Fig. 4, most of the oceanographic structure is expected to be seen in the first coefficient.
Fig. 5. The acoustic fitness function, $P(T_{\Delta SE})$, is computed for ten different groups of 1000 SSPs using a threshold $T_{\Delta SE} = 6dB$. The SSPs are generated using (8) with the EOFs and eigenvalues estimated from the data set presented in section III, with one minor exception: The last group utilizes the actual eigenvalues as the variances in (8), but in the remaining groups the variances are lowered in 10% steps. E g the first group uses a variance 90% lower than the last group.

C. Results

The described method for determining acoustic stability is applied on the data sets described in section III. The acoustic model Lybin [30] is used to model the signal excess. A towed array sonar at 50 m depth and working at frequencies around 1.5 kHz is used in the acoustic modelling. The bottom is assumed flat and 300 m deep. A group of SSPs is deemed acoustically stable if AFF is greater than 90%. The threshold on signal excess variations, $T_{\Delta SE}$, see (12), is set to 5 dB. The stability analysis is limited to ranges between 2 km and 10 km, and depths between 20 m and 200 m. All locations within the area are considered equally probable target locations. Signal excess levels greater than 0 dB are required, see (15).

Figs. 8 and 10 show the geographical distribution of acoustically stable groups for the full and subsampled data set and subset 1. Table I lists statistical data for each combination of data set and subdivision algorithm. Due to
Fig. 6. The estimated standard deviation of the probability \( P(T_{\Delta SE}) \) in percent as a function of the number of SSPs used to estimate \( P(T_{\Delta SE}) \). A group of 1000 SSPs are generated using (8), where \( \kappa_k \) are modelled as zero–mean Gaussian random functions with variances given by \( \lambda_k \). The standard deviations are estimated from random subsets of this group. Twenty subsets per subset size are used. The subset size is the number of SSPs used to estimate \( P(T_{\Delta SE}) \).

Computing power limitations on the number of SSPs, the CC method was not used on the full data set. The entire subset 2 data set was in fact judged to be acoustically stable, with AFF exceeding 90% for all SSPs in that area.

We compare the distribution of groups for the full (Fig. 8 a) and subsampled (Fig. 8 b) data sets when using the OCM subdivision algorithm. The similarities for the largest groups (yellow, red, pink, light green and dark green colors) are obvious, both in area distribution and shape similarities. As for the smaller groups, their speckled, non-contiguous distribution indicate areas of high acoustic variability. It is expected that these small speckled groups are highly sensitive to the resolution of the data set, and detailed study of these discrepancies does not reveal much of interest. However, it is interesting to note that unclassified areas – the white areas in Fig. 8 a) and b) – are much smaller in the subsampled data set. These areas contain SSPs in groups that fail to meet the minimum size criteria after subdivision (section II).
Fig. 7. The first EOF coefficient is plotted with surface salinity contours on top. The value of the surface salinity contour is 34.5 psu. Areas with surface salinity values above 34.5 psu are assumed AW, and the remaining areas are considered CW.

The most distinct differences between the OCM (Fig. 8 b) and CC results (Fig. 8 c) is that fewer, more contiguous, and larger groups are generated when using the CC method. There are no longer any voids due to groups that fail to meet the minimum size criteria. The CC method on the subsampled data set (Fig. 8 c) divides the whole area into four groups, much less than OCM subsampled (15 groups) and OCM full data set (10 groups). The shape and area of the largest OCM groups are comparable to the CC groups: The yellow areas are strikingly similar and the light green area of the CC method corresponds to the light green and red areas of the OCM full and subsampled data set. The pink CC area is composed of the pink, brown and dark green areas of the OCM method. There are some nuances in the exact boundaries, particular in the OCM areas of high variability, but the similarities in shape and extent are striking. OCM generates more groups because acoustically unstable groups are split into several groups, depending on the number of coefficients used. The CC subdivision algorithm, on the other hand, splits an acoustically unstable group in two.

Fig. 9 shows the sound speed as a function of depth along the east–west cross-section at 60.5 N (upper panel) together with the group distribution from the OCM and CC method along this line (lower panel). The classical
Fig. 8. The group distributions for the large area data sets. (a) Full resolution, OCM method. (b) Reduced resolution, OCM method. (c) Reduced resolution, CC method. Each colour represents a different group of SSPs. The black rectangles show the location of subset 1 (60-60.17 N) and subset 2 (60.47-60.92 N).

A wedge-like structure of CW close to the coast can be seen, corresponding to systematic lower sound speed values in the upper right part of the cross section. The associated changes in vertical sound speed gradient and how that affects the sonar performance is distilled in the distribution of groups. The yellow group is clearly AW, with moderate vertical sound speed gradients. Further east, the green group of the CC method and the red group of the OCM method is associated with a slab of CW (low sound speed) above AW, and with a stronger vertical sound speed gradient at the transition between those two water masses. The interface is tilted downwards to the east. Closest to the coast, the pink (CC) and brown (OCM) group classifies a set of vertical SSPs where the interface between CW and AW is deeper, the gradient is stronger and the location of the strongest gradients are shifted downwards. The fact that the CC method produces fewer and more contiguous groups than the OCM method is also apparent in this cross section.

Subsets 1 and 2 are subsets of the full data set. Subset 1 is placed in an area with high concentration of groups when processing the full and subsampled data set, see Fig. 8 and 10. The high concentration of groups indicates
that this area is acoustically unstable, see table IV-C.

Subset 2 is placed in an area with low concentration of groups when processing the full and downsampled data set. Processing the SSPs contained in subset 2 resulted in a single group with an AFF of 93% and therefore well above the threshold. This shows that groups initially classified as stable areas, see Fig. 8, are classified as stable also when applying the full method on SSPs from that area only. Note that when applying the method on the full data set, there is a second group also in the area containing subset 2, this group is not present when applying the method on data from subset 2 only because the group is too small to reduce the AFF below the 90% threshold.

Figs. 8 and 10 may serve as stability maps for sonar and rapid environmental assessment operations. A sonar platform operating in the presented area is advised to measure the SSP more frequently in areas with small, non–contiguous groups, e. g. subset 1. In contrast, less frequent measurements are required in areas like subset 2, where all profiles are contained in a single acoustically stable group. Similarly, in rapid environmental assessment missions more resources must be allocated to unstable regions than stable regions. Maps such as these may therefore provide valuable assistance in estimating how often new SSP measurements should be undertaken.
Fig. 10. The group distributions for sub area 1 using the OCM (left) and CC method (right).

Another useful application is for sonar operation planning. Detection ranges, based on representative SSPs from each group, may be estimated. The representative SSPs are generated by inputting the groups median coefficients in (8). Since the acoustic model is run only once per group, different sets of sonar parameters may be tested in order to maximise the detection range in each group at minimum computational cost. Note that the initial grouping is based on a single set of sonar parameters. Changes in the sonar parameters influence the groups acoustic stability, but studies made in connection to the presented work have shown that the group distribution remains almost the same for small changes in the sonar parameters. The considered sonar parameters were source level and sonar depth. This study is not considered within the scope of this work and therefore not presented here. If entirely different sonar systems are considered, for instance sonars working on different frequencies, then the method must be repeated for the specific sonar system. Furthermore, the method can easily be extended to other sonar applications not involving sonar performance prediction, but in that case the acoustic fitness function should be reconsidered.

V. SUMMARY

A set of modelled sound speed profiles (SSP) is represented by empirical orthogonal functions (EOF). An approach based on EOF coefficients is shown equally efficient in classifying different water masses as traditional approaches based on salinity values. The EOF based approach is refined into two different automatic algorithms for separating groups of SSPs into several lesser and more homogeneous groups of SSPs; clustering of coefficients (CC) and ordering of coefficient magnitudes (OCM). The algorithms are combined with an acoustic fitness function, which is used for evaluating the acoustic stability within a group of SSPs. The combined method efficiently splits a large set of SSPs into several smaller acoustically stable groups.

The proposed method outputs a map of the group structure. Such maps may be used as decision aids for how often SSPs should be measured in a given area. Stable regions typically consist of large and geographically contiguous groups. Regions containing many small and non-contiguous groups are typically acoustically unstable, meaning that
the acoustic field is sensitive to the present oceanographic variations. For instance, during a rapid environmental assessment mission using gliders, the concentration of gliders should be higher in unstable regions than in stable regions. Likewise, during sonar operations, SSPs should be measured more frequently in unstable regions, than in stable regions.

Another possible application, useful for planning sonar operations, is to present maps of detection ranges based on a representative SSP from each group. A single representative SSP for an entire stable area also allows efficient optimisation of sonar performance. Sonar performance is modelled for the representative SSP. The optimisation is performed by tuning the sonar parameters for maximum sonar performance.

The method may be tuned to a specific type of operation by varying a set of parameters. The parameters include a priori knowledge of target behavior, for instance the maximum and minimum target depth. Other parameters allows the user to tune the requirements for a group of profiles to be classified as acoustically stable.

REFERENCES


TABLE I
RESULTS FROM ALL COMBINATIONS OF DATA SETS AND SUBDIVISION ALGORITHMS.

<table>
<thead>
<tr>
<th>Data set</th>
<th>Method</th>
<th>No groups</th>
<th>No profiles</th>
<th>Mean AFF</th>
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</thead>
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<tr>
<td>Full data set</td>
<td>OCM</td>
<td>10</td>
<td>8816 (88%)</td>
<td>94%</td>
</tr>
<tr>
<td>Subsampled data set</td>
<td>OCM</td>
<td>15</td>
<td>2715 (95%)</td>
<td>94%</td>
</tr>
<tr>
<td></td>
<td>CC</td>
<td>4</td>
<td>2873 (100%)</td>
<td>93%</td>
</tr>
<tr>
<td>Subset 1</td>
<td>OCM</td>
<td>5</td>
<td>600 (83%)</td>
<td>94%</td>
</tr>
<tr>
<td></td>
<td>CC</td>
<td>3</td>
<td>640 (89%)</td>
<td>94%</td>
</tr>
<tr>
<td>Subset 2</td>
<td>Both</td>
<td>1</td>
<td>All</td>
<td>93%</td>
</tr>
</tbody>
</table>
Paper 7

In ocean evaluation of low frequency active sonar systems
In ocean evaluation of low frequency active sonar systems

Karl Thomas Hjelmervik
Geir Helge Sandsmark

Sonar performance measurements in the sea are always affected by uncontrollable and/or uncertain environmental conditions as sound speed variations, bottom topography or the acoustic properties of the sea floor. This paper presents a method to determine a sonar – target geometry which minimizes the uncertainty in target signal excess due to environmental variability.

An acoustic model is used to estimate the signal excess for a large number of sound speed profiles measured in the relevant area. The results are compared while searching for a target range and depth where the estimated signal excess is robust with respect to the expected variability of the sound speed profile in the actual area. Results from sea trials will be presented, as well as simulated examples used to demonstrate the achieved robustness or sensitivity of the signal excess to environmental changes, at different test geometries.

1 Introduction

In general there is a need for quantifiable sonar performance tests carried out at sea under conditions resembling the normal working conditions for the equipment. The accuracy and reliability of such tests are frequently questioned. The limited confidence in such tests is due to acknowledged uncertainty in the environmental parameters and experienced inaccuracy of meso-scale ocean acoustic experiments. However experimental verification of propagation models may often show good agreement for some measurements while under different conditions there is virtually no resemblance between model and reality. Also a closer look at some modelling results indicates that the sensitivity of the received signal level, to for instance the target location, may vary significantly over an actual area in the ocean. In some cases the signal excess may remain near constant over a significant depth and range interval, while only a few meters displacement may cause large deviations of the signal or reverberation levels. Similarly a small deviation in the sound speed profile may cause entirely different propagation patterns for some cases and experiment geometries while other choices of target and sonar locations may provide more robust conditions.

With this background we have developed a method of conducting experiments at sea where the uncertainty of the results are limited, quantifiable and assessable. The method is based on running an acoustic model repeatedly. Each run uses a single sampled realisation of the environment as input. This paper focuses on variations in sound speed profile. Therefore a sample sound speed profile represents a realisation of the environment. The computations are based on a selection of sound speed profiles measured within the actual area as close to the test schedule as possible. The results are then analyzed to find favourable positions for the sonar and the target.

Overall considerations and aspects of underwater sensor testing is presented in ref (1). The current paper goes into more detail on how to handle the acoustic sensitivity issues due to varying oceanography.

Section 2 presents the tools used in the analysis. Section 3 shows an example of how the method can be used to find good locations for the sonar and the target during a test. Section 5 concludes the paper.

2 Numerical tools

Two different numerical tools are used in the method of finding stable conditions for testing acoustic equipment. The main tool is Lybin, an acoustic ray trace model that estimates the signal excess in a single vertical cross-section for a given environment and sonar. The second tool is a method of presenting the sensitivity of the signal excess to environmental variation. The results are presented graphically, denoted “stability plots”.

Lybin

Lybin is an acoustic ray trace model developed by Svein Mjølsnes, Norwegian Defence Logistic Organization. Ref (2) gives an overview of ray tracing and the underlying theory. The model is two-dimensional, covering depth and range. It estimates the transmission loss, the reverberation level and the noise level based on sonar data and environmental data. These data are applied to the sonar equations for estimation of signal excess. Detection theory (5,6) is used to find the probability of detection and the corresponding detection range. In this paper the signal excess is used.

Lybins transmission loss module was verified by NURC\(^1\), (3). The evaluation team conclude: “The general conclusion of this test is that the range-dependent ray-trace model LYBIN, developed by the Norwegian Navy, is indeed a valid alternative to

\(^1\) NATO Underwater Research Centre
existing propagation models in the AESS\textsuperscript{2}. The LYBIN model has prediction accuracy similar to the GRAB ‘reference’ model but is considerably faster.”

Lybin was presented at the Underwater Defense Conference and Exhibition in Glasgow 2008, ref (4).

Stability plots

The idea of stability plots is to compare the signal excess results from several different Lybin-runs, and find ranges and depths where the signal excess remains nearly constant from run to run.

For the purpose of finding stable acoustic conditions in an area of both spatially and temporally varying oceanography, Lybin is run using several different sound speed profiles measured in the actual area. The results from these runs are then compiled into the stability plots. One of these sound speed profiles is selected as a representative sound speed profile for the entire set of sound speed profiles. This could either be a mean of all the other sound speed profiles, or presumably better, a single measured sound speed profile, possessing some characteristics judged as typical for that set of sound speed profiles. In the latter case, the representative sound speed profile should be smoothed to remove measurement artifacts.

The stability plot shows the stability parameter, $SP$, given by:

$$SP(r,z) = \frac{1}{N} \sum_{i=1}^{N} \text{step}\left(T - \left|SE_i(r,z) - SE(r,z)\right|\right)$$

$SE_i(r,z)$ is the modelled signal excess at range $r$ and depth $z$, measured in dB, using the representative sound speed profile. $SE(r,z)$ is the modelled signal excess for the i-th sound speed profile in the set. $N$ denotes the size of the set. $T$ is a set threshold, for example 3dB as used in this work. $\text{step}(\cdot)$ denotes the unit step function taking the value 1 for positive arguments and 0 elsewhere. $SP(r,z)$ is therefore a two-dimensional matrix of values between 0 and 1. The value of a single element is simply the fraction of cases that has a signal excess deviation from the typical case, lower than the selected threshold. Thus, an element takes the value 1 if the complete set of sound speed profiles results in a signal excess difference less than the selected threshold. The value 0.5 indicates that half the set of sound speed profiles results in a low signal excess difference. Figure 2 shows an example of the stability plot. The red areas represent areas where 100% of the runs resulted in signal excess values within a margin of $T$ from the signal excess computed using the representative sound speed profile. Simply stated, red areas are stable, blue areas are unstable.

3 Results

The task of finding a stable environment for testing of the acoustic equipment is divided into two parts. First, a historically stable area must be found. Areas prone to oceanographic fronts or strong variations in terrain should be avoided. Second, just prior to the testing of the equipment, oceanographic measurements should be made to find the most stable region in that area and the relative positions of the equipment resulting in the most stable conditions. This paper is focused on the second part.

In the present example, a monostatic sonar is tested using a stationary, artificial test target (echo repeater). During the test, the distance between the sonar and the target is kept constant by letting the sonar vessel encircle the target. Three geometric parameters remain to be determined, in order to gain stable conditions for the test: sonar depth and target depth and range (distance from sonar vessel).

Sound speed measurements

The sound speed profiles used in this study was measured in November 2007, along lines using a towed CTD. The ten lines were approximately 27km long with 5km separation between the lines, see figure 3. Each star in figure 3 corresponds to a single sound speed profile. The red stars indicate positions suitable for performing the acoustic tests due to homogenous sound speed profiles. In the following analyses these seven sound speed profiles are used. The measurements resulted in a total of 170 sound speed profiles. Figure 4 presents all the measured sound speed profiles. Figure 5 shows a filled contour plot as a function of range and depth for line nr 5.

\textsuperscript{2} Allied Environmental Support System
4.1 Positions of the sound speed measurements made. The coordinates are presented in decimal degrees. The sound speed profiles were measured along east-west oriented lines.

Figure 3

4.2 Figure 4 170 sound speed profiles. Notice the strong variations below 60m depth compared to above 60m depth. The red curves are the sound speed profiles measured in the positions indicated by the red stars in figure 3. The yellow curve depicts the selected and smoothed representative sound speed profile.

4.3 Figure 5 Sound speed as a function of depth and range from west to east along line 5 (Line one in figure 3 is furthest to the north). The preferred area is between the two black vertical lines.

4.4 Stability plot

The threshold, \( T \), for determining the stability parameter was set to 3dB. Figure 6 shows a stability plot for the sonar at 50m depth. The red areas indicate range - depth pairs with stable signal excess, and therefore suitable positions for the target. Figure 7 shows the stability picture when the sonar is at 5m depth. Both cases show reasonably large areas for robust measurements. This is however not always the case. Figure 8 shows a stability plot using a set of sound speed profiles measured in April 2008. In this case the target should be deeper than 60m in order to ensure stability.

Figure 6 Stability plot for a sonar at 50m depth.

Figure 7 Stability plot for a sonar at 5m depth.
Figure 7 Stability plot for a sonar at 5m depth.

Figure 8 Stability plot for a sonar at 25m depth.

5 Conclusion

A mono-variable perturbation analysis is used to quantify the sensitivity of sonar performance measurements to temporal and spatial variations of environmental parameters with impact on the sonar performance. This analysis indicates that by a careful choice of sonar and target deployment, the sensitivity to unaccounted parameter variations may be kept within acceptable limits. Stability plots are introduced to quantify the stability of different sonar-target geometries.

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References


